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Task Scheduling Problem Using Fuzzy Graph

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ABSTRACT

The concept of obtaining fuzzy sum of fuzzy colorings problem has a natural application in scheduling theory. The problem of scheduling N jobs on a single machine and obtain the minimum value of the job completion times is equivalent to finding the fuzzy chromatic sum of the fuzzy graph modeled for this problem. The aim of this paper is to solve task scheduling problems using fuzzy graph.

Key-words: Fuzzy Graph, K-Fuzzy Coloring, Chromatic Fuzzy Sum of Graph, Γ-Chromatic Sum of Graph.

INTRODUCTION

The field of mathematics plays vital role in various field. One of the important areas in Mathematics is graph theory, which is used in several models. The origin of graph theory started with Konigsberg bridge problem, in 1735. It was a long standing problem until solved by Leonhard Euler, by means of graph. The colouring problem consists of determining the chromatic number of a graph and an associated colouring function. Let G be a simple graph with n vertices. A colouring of the vertices of G is a mapping f: V (G) → N, such that adjacent vertices are assigned different colours. The chromatic sum of a graph introduced in[5] is defined as the smallest possible total over all vertices that can occur among all colourings of G. Senthilraj S.[10] generalize these concepts to fuzzy graphs. He define fuzzy graphs with fuzzy vertex set and fuzzy edge set and generalize the concept of the chromatic joins and chromatic sum of a graph to fuzzy graphs and define the fuzzy chromatic sum of fuzzy graph. Author consider the problem of scheduling N jobs on a single machine and obtain the minimum value of the job completion times which is equivalent to finding the fuzzy chromatic sum of the fuzzy graph modeled for this problem by considering the example of scheduling 6 jobs on a single machine.

In this paper we generalize the above said result by considering the case of scheduling 8 tasks on a single machine and obtain a minimum value of the task completion time.

Preliminaries and Results

Definition 2.1:[11]

A fuzzy set A defined on a non empty set X is the familyA = $\{(x,\mu_A(x))/x \epsilon X\}$, where $\mu_A : X \rightarrow I$ is the membership function. In classical fuzzy set theory the set I is usually defined as the interval [0,1] such that $\mu_A(x) = \begin{cases} 0 : x \notin X \\ 1 : x \in X \end{cases}$

It takes any intermediate value between 0 and 1 represents the degree in which$x \epsilon A$. The set I could be discrete set of the form $I = \{0,1,\ldots k\}$ where $\mu_A(x) < \mu_A(x')$ indicates that the degree of membership of x to A is lower than the degree of membership of x'.

Definition 2.2

Let V be a finite nonempty set. The triple $\hat{G} = (V,\sigma,\mu)$ is called a fuzzy graph on V where σ and μ are fuzzy sets on VandE, respectively, such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $u,v \epsilon V$ anduv ϵE. For fuzzy graph $\hat{G} = (V, \sigma,\mu)$, the elements V and E are called set of vertices and set of edges of G respectively.

Definition 2.3

A fuzzy graph $\hat{G} = (V,\sigma,\mu)$ is called a complete fuzzy graph if $\mu(uv)= \sigma(u) \wedge \sigma(v)$ for all $u,v \epsilon V$ and$uv \epsilon E$. We denote this complete fuzzy graph by $\hat{G}\ k$.

Definition 2.4

Two vertices u and v in $\hat{G}$ are called adjacent if (½)[$\sigma(u) \wedge \sigma(v)$] $\leq \mu(uv)$.

Definition 2.5

The edge uv of $\hat{G}$ is called strong if u and v are adjacent. Otherwise it is called weak.

Definition 2.6

A family $\Gamma = \{Y_1, Y_2, \ldots.., Y_k\}$ of fuzzy sets on V is called a k-fuzzy coloring of $\hat{G} = (V,\sigma,\mu)$ if

a) $\wedge\Gamma = \sigma$

b) $Y_i \wedge Y_j = 0$

c) For every strong edge uv of $\hat{G}, Y_i(u) \wedge Y_j(v) = 0$ *for* $1 \leq i \leq k$.

The above definition of k-fuzzy coloring was defined by the authors Eslahchi and Onagh [1] on fuzzy set of vertices.

Definition 2.7[10]

The least value of k for which Ghas a fuzzy coloring, denoted by$x^f(G)$, is called the fuzzy chromatic number of G.

Definition 2.8

For a k-fuzzy coloring $\Gamma = \{Y_1, Y_2, \ldots.., Y_k\}$ of a fuzzy graph of$G$, Γ chromatic fuzzy sum of G denoted by $\sum_\Gamma(G)$is defined as

$$\Sigma_\Gamma(G) = 1\sum_{x\in C_1}\theta_1(x) + 2\sum_{x\in C_2}\theta_2(x) + \cdots \ldots. + k\sum_{x\in C_k}\theta_k(x)$$

Where C_i=$supp_i$ and $\theta_i(x)$=max σ {(x)+$\mu(xy)$/y ϵ C_i}.

Definition 2.9

The chromatic fuzzy sum of G denoted by $\sum(G)$is defined as follows

$\sum(G)$=min $\{\sum_\Gamma(G)/\Gamma$ is fuzzy colouring}.

The number of fuzzy coloring of G is finite and so there exist a fuzzy Γ_0 which is called minimum fuzzy coloring of G such that $\sum(G)=\sum_{\Gamma 0}(G)$.

Theorem 2.1

Let G be a fuzzy graph and Γ_0= $\{Y_1, Y_2, \ldots.., Y_k\}$ is minimum fuzzy sum coloring of G.Then $\sum_{x\epsilon C1} \theta_1(x) \geq \sum_{x\epsilon C2} \theta_2(x) \geq \cdots \ldots \geq \sum_{x\epsilon CK} \theta_K(x)$

Theorem 2.2

For a fuzzy graph $\hat{G} = (V,\sigma,\mu)$,$\sum(G)\leq 3/4[(x^f(G)+1)h(\sigma)|V|$, where $h(\sigma)$is height of σ and$|V|$is cardinality of V.

Remarks[10]

Let $\hat{G} = (V,\sigma,\mu)$be a connected fuzzy graph with estrong edges. Then the lower bound for $\sum(G)$is $w\sqrt{8e}$, where w=max{$\sigma(x)+\mu(xy)>o, x \epsilon V,(x,y)$is weak edge of G}.

The fuzzy chromatic sum lies between $w\sqrt{8e}$and$3/4[(x^f(G)+1)h(\sigma)|V|$.

RESULTS AND DISCUSSIONS

Result 3.1

Find a minimum value of the task completion time for scheduling 8 tasks on a single machine.

Assume that at any time the machine is capable to perform any number of tasks and these tasks are independent or conflicts between them are less than one. Consider the time consuming
for task 1and 4 is 0.4hrs,
for tasks 3 and 6 is 0.3hrs,
for tasks 2 and 5 is 1hrs.
for tasks 7 and 8 is 0.2hrs.

Also,

Task {(2, 5), (5, 6), (6, 7)}conflict together with 0.1 hrs.
Task, {(1, 2), (1, 4), (1, 5), (2, 4), (4,7), (4, 8)}conflict together with 0.4 hrs.

Task {(1, 3), (2, 8), (3, 4), (4, 5), (5, 7)}conflict together with 0.3 hrs.

Now, we define the fuzzy graph for above problem.

Let $\hat{G} = (V, \sigma, \mu)$ where V is the set of all task, $\sigma(x)$ is the amount of consuming time of machine for each $x \in V$ and $\mu(x,y)$ is the measure of the conflict between the task x andy. Finding the minimum value of job completion time for this problem is equivalent to the chromatic sum of $\hat{G}$.

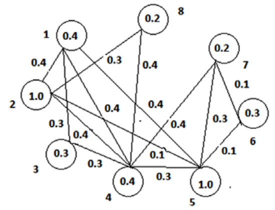
The fuzzy graph $\hat{G} = (V, \sigma, \mu)$ corresponding to our example is defined as follows:

Let V={1,2,3,4,5,6,7,8},

$$\sigma(i) = \begin{cases} 0.4 \ for\ i = 1,\ 4 \\ 1.0 \ for\ i = 2,5 \\ 0.3 \ for\ i = 3,6 \\ 0.2 \ for\ i = 7,8 \end{cases}$$

$$\mu(i,j) = \begin{cases} 0.1, & for\ i,j = \{(2,5),(5,6),(6,7)\} \\ 0.4, & for\ i,j = \{(1,2),(1,4),(1,5),(2,4),(4,7),(4,8)\} \\ 0.3, & for\ i,j = \{(1,3),(2,8),(3,4),(4,5),(5,7)\} \end{cases}$$

The fuzzy graph for above problem is



Strong edges: (1, 2), (1,3), (1, 4), (1, 5), (2, 4), (3, 4), (4, 5), (4, 7), (4, 8), (5, 7), (6, 7).

Weak edges: (1, 6), (1, 7), (1, 8), (2, 3), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 6), (5, 6), (5, 8), (6, 8), (7, 8).

Let $\Gamma_1 = \{Y_1, Y_2, \ldots, Y_8\}$ be a family of fuzzy set defined on V where

$$\gamma_1(i) = \begin{cases} 1.0, & for\ i = 2 \\ o, & otherwise \end{cases}, \gamma_2(i) = \begin{cases} 1.0, & for\ i = 5 \\ o, & otherwise \end{cases}, \gamma_3(i) = \begin{cases} 0.4, & for\ i = 1 \\ o, & otherwise \end{cases}$$

$$\gamma_4(i) = \begin{cases} 0.4, & for\ i = 4 \\ o, & otherwise \end{cases}, \gamma_5(i) = \begin{cases} 0.3, & for\ i = 3 \\ o, & otherwise \end{cases}, \quad \gamma_6(i) = \begin{cases} 0.3, & for\ i = 6 \\ o, & otherwise \end{cases}$$

$$\gamma_7(i) = \begin{cases} 0.2, & for\ i = 7 \\ o, & otherwise \end{cases}, \ \gamma_8(i) = \begin{cases} 0.2, & for\ i = 8 \\ o, & otherwise \end{cases}$$

V	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	max $Y_1 = \sigma$(i)
1	0	0	0.4	0	0	0	0	0	0.4
2	1.0	0	0	0	0	0	0	0	1.0
3	0	0	0	0	0.3	0	0	0	0.3
4	0	0	0	0.4	0	0	0	0	0.4
5	0	1.0	0	0	0	0	0	0	1.0
6	0	0	0	0	0	0.3	0	0	0.3
7	0	0	0	0	0	0	0.2	0	0.2
8	0	0	0	0	0	0	0	0.2	0.2

From the above table, we can see that Γ_1 satisfied all the properties of k- fuzzy coloring.

Therefore G has 8-Coloring and $x^f(G)$=8. For this 8-Coloring, Γ_1 chromatic number can be calculated as follows:

C_1={3,5}.C_2={2}, C_3={1, 7, 8}, C_4={4}, C_5={3}, $\theta_1(3)$=max{0.3+0,0.3+0}=0.3, $\theta_1(5)$=max{1+0,1+0}=1, $\theta_2(2)$ = max{1+0} = 1, $\theta_3(1)$ = max{0.4+0,0.4+0,0.4+0}=0.4, $\theta_3(7)$=max{0.2+0,0.2+0,0.2+0}=0.2,$\theta_3(8)$=max{0.2 +0,0.2+0,0.2+0}=0.2, $\theta_4(4)$=max{0.4+0}=0.4,$\theta_5(6)$=max{0.3+0}=0.3
The Γ_2 chromatic fuzzy sum of G

$\sum_{\Gamma 2}(G)$=1(1+0.3)+2(1)+3(0.4+0.2+0.2)+4(0.4)+5(0.3)=8.4

Let $\Gamma_3 = \{Y_1, Y_2, Y_3\}$ be a family of fuzzy set defined on $V$where

$$\gamma_1(i) = \begin{cases} 1.0, & for\ i = 2,5 \\ 0.3, & for\ i = 3,6 \\ o, & otherwise \end{cases}, \gamma_2(i) = \begin{cases} 0.4, & for\ i = 1 \\ 0.2, & for\ i = 7,8 \\ o, & otherwise \end{cases}, \gamma_3(i) = \begin{cases} 0.4, & for\ i = 4 \\ o, & otherwise \end{cases}$$

V	Y_1	Y_2	Y_3	$maxY_1 = \sigma$**(i)**
1	0	0.4	0	0.4
2	1.0	0	0	1.0
3	0.3	0	0	0.3
4	0	0	0.4	0.4
5	1.0	0	0	1.0
6	0.3	0	0	0.3
7	0	0.2	0	0.2
8	0	0.2	0	0.2

Again from the above table, we can see that Γ_3 satisfied all the properties of k- fuzzy coloring.

Therefore *G* has 3-Coloring and $x^f(G)$=5. For this 3-Coloring, Γ_3 chromatic number can be calculated as follows:

C_1 = {2,3,5,6}, C_2 = {1,7,8}, C_3 = {4}
θ_1(3)=max{0.3+0,0.3+0,0.3+0}=0.3,
θ_1(5)=max{1+0.1,1+0,1+0,1+0.1=1.1,
θ_1(2)=max{1+0,1+0,1+0.1,1+0}=1.1,
θ_1(6)=max{0.4+0,0.4+0,0.4+0.1,0.4+0}=0.5,
θ_2(7)=max{0.2+0,0.2+0,0.2+0}=0.2,
θ_2(8)=max{0.2+0,0.2+0,0.2+0}=0.2,
θ_2(1)=max{0.4+0,0.4+0,0.4+0}=0.4,
θ_3(6)=max{0.3+0}=0.3

The Γ_3 chromatic fuzzy sum of G

$\sum_{\Gamma 3}(G)$=1(1.1+1.1+0.5+0.3)+2(0.2+0.2+0.4)+3(0.3)=5.5

Therefore the fuzzy chromatic sum of G is

$\sum(G)$=min{$\Gamma_1, \Gamma_2, \Gamma_3$}

=min{12.1, 8.4, 5.5}=5.5

Calculation for w
w= min{0.4+0,0.4+0,0.4+0,1+0,1+0.1,1+0,1+0,0.3+0,0.3+0,0.3+0,0.3+0,0.4+0,1+0,1+0.1,1+0,0.3+0,0.2+0}=0.2

Lower bond of $\sum(G)$ is $w\sqrt{8e}$=0.2 8*12=0.2*4*√6=1.9592

Now,

3/4[($x^f(G)$+1)*h*(σ)|*V*|=3/4 [(4+1)×1×8]=30

CONCLUSION

The fuzzy chromatic number lies between 30 and 1.9592. In our problem $\sum(G)$=5.5

Therefore the minimum time of task completion of our problem is 5.5hrs.

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