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# An Improved Ratio-Type Variance Estimator by Using Linear Combination of different Measures of Location

**MUHAMMAD ALI, MUHAMMAD KHALIL, MUHAMMAD HANIF, NASIR JAMAL and USMAN SHAHZAD***

Pir Mehr Ali Shah Arid Agriculture University Rawalpindi, Pakistan.

**Abstract**

In this research study, modified family of estimators is proposed to estimate the population variance of the study variable when the population variance, quartiles, median and the coefficient of correlation of auxiliary variable are known. The expression of bias and mean squared error (MSE) of the proposed estimator are derived. Comparisons of the proposed estimator with the other existing are conducted estimators. The results obtained were illustrated numerically by using primary data sets. Theoretical and numerical justification of the proposed estimator was done to show its dominance.

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## Introduction

In our everyday life variations are available all over the place. It is the idea of law that people or no two things are precisely same. For example, an agriculturist needs a sufficient comprehension of the varieties in climatic factors particularly from place to place (or time to time) to have the capacity to anticipate when, how and where to plant his yield. For consistent information of the level of variations in individuals' response a maker need to lessen or increment cost of his item, or make strides the nature of his item. A doctor needs a full comprehension of variations in the body temperature, level of human circulatory strain and heartbeat rate for full medicine. Estimating of these restricted population variance (variation) has enormous significance in various fields such as manufacturing, cultivation, health and natural sciences where we come across the populations which are expected to be skewed.

Variation is at hand everywhere in our day to day life. It is law of natural world that no two things or individuals are closely alike. For instance, a medical doctor needs a full understanding of dissimilarity in the degree of human blood stress, body temperature and beat rate for sufficient prescription(Singh 2005).

**CONTACT** Usman Shahzad usman.stat@yahoo.com Pir Mehr Ali Shah Arid Agriculture University Rawalpindi, Pakistan.

**Simple Random Sampling with out Replacement Sample Vriance**

In the case of simple random sampling without replacement sample variance $s_y^2$ is used to estimate the population variance $s_y^2$ which is an unbiased estimator and variance is given below:

$$v(s_y^2) = \gamma s_y^4 (\beta_{2(y)} - 1)$$

**Ratio type Estimation for Estimation of Population Variance**

Isaki (1983) planned the ratio type variance estimator for the population variance $s_y^2$ when the population variance $S_x^2$ of the auxiliary variable X is known the estimator together with its bias, mean square error given below:

$$\hat{\tau}_i^2 = s_y^2 [S_x^2/(S_x^2]$$

$$\text{Bias} = \gamma s_y^2 \Delta_i [\Delta_i (\beta_{2x}-1) - (\theta-1)]$$

$$\text{Mean Squared Error} = \gamma s_y^4 [(\beta - 1) + \Delta_i^2 (\omega - 1) - 2\Delta_i (\theta-1)]$$

Where Constant,

$$\Delta_i = (S_x^2) / (S_x^2 + N_j )$$

The ratio type variance estimator used to improve the precision of the estimate of the population variance compared to SRSWOR sample variance. Further improvements are also achieved on the ratio estimator by introducing a number of modified ratio estimators with the use of known parameters like Median, Quartiles and Coefficient of correlation. The problem of constructing efficient estimators for the population variance had been widely discussed. For the purpose of this study we reviewed the estimators developed by Subramani and Kumarapandiyan, (2012a, 2012b, 2012c) in Table 1. Further, interested readers see; Shahzad (2016) and Shahzad *et al*. (2017).

**Material and Methods**

Assume a sample with size n from a population with size N, selected by a precise sampling design. Let Y be the variable which is the entity of study and X, the available auxiliary variable. For a condition in which the population means, X is available, some estimators of the population variance Y had been planned. We have considered variance ratio method, for estimating a population variance. Selecting sample according to simple random sampling, and we have proposed a general class of estimators. The presentation properties of the planned estimators are analyzed with respect to the bias, mean squared error criteria using asymptotic theory, and we find the most favorable values in each planned class. The planned estimators are legitimated, advanced on the usual estimators reducing the errors obtained.

**Notations**

The following notation are used for numerical illustrations

- N - Population Size
- n - Sample Size
- $\gamma = (1-f)/n$
- Y - Study Variable
- X - Auxiliary Variable
- $C_x$ - Coefficient of variation of x
- $C_y$ - Coefficient of variation of y
- $M_d$ - Median of the auxiliary variable
- $Q_1$ - First Quartile of the auxiliary variable
- $Q_2$ - Second Quartile of the auxiliary variable
- $Q_3$ - Third Quartile of the auxiliary variable
- $Q_d$ - $(Q_3 - Q_1)/2$ Quartile Deviation of the auxiliary variable
- $Q_r$ - $Q_3 - Q_1$ Range Quartile of the auxiliary variable
- $Q_a$ - $(Q_3 + Q_1)/2$ Sum of first and third Quartile of the auxiliary variable
- $\beta_{1x}$ - Skewness of the auxiliary Variable
- $\beta_{2x}$ - Kurtosis of the auxiliary Variable
- $\beta$ - Kurtosis of the study Variable
- $S_x$ - Standard deviation of the auxiliary variable
- $S_y$ - Standard deviation of the study variable
- MSE (.)- Mean Squared Error
- $\rho$ – Coefficient of Correlation
- Bias (.) Bias
- $\hat{\tau}_{Nj}^2$ - Variance Ratio Estimator
- $\lambda_{rs} = \mu_{rs} / \mu^{r/2}{}_{20} \mu^{s/2}{}_{02}$
- $\mu_{rs} = 1/N \sum^N{}_i = 1 (Y_i - \bar{Y})(X_i - \bar{X})$
- $\Delta_i$ - constant of the existing and proposed estimator
- $N_j$ - linear combination of the existing and proposed estimator proposed class.

**Table 1: Bias, Mean Squared error of the Existing Estimators**

| **Estimators** | **Bias B (.)** | **Mean Squared Error MSE (.)** |
|---|---|---|
| $\hat{\tau}_1^2 = s_y^2\left[\frac{\beta_{1x}S_x^2 + \beta_{2x}}{\beta_{1x}s_x^2 + \beta_{2x}}\right]$ J.Subramani* | $\gamma s_y^2 \Delta_1 [\Delta_1 (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_1^2 (\beta_{2x}-1) - 2\Delta_1 (\theta-1)]$ |
| $\hat{\tau}_2^2 = s_y^2\left[\frac{\beta_{1x}S_x^2 + \rho}{\beta_{1x}s_x^2 + \rho}\right]$ J.Subramani* | $\gamma s_y^2 \Delta_2 [\Delta_2 (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_2^2 (\beta_{2x}-1) - 2\Delta_2 (\theta-1)]$ |
| $\hat{\tau}_3^2 = s_y^2\left[\frac{\beta_{1x}S_x^2 + S_x}{\beta_{1x}s_x^2 + S_x}\right]$ J.Subramani* | $\gamma s_y^2 \Delta_3 [\Delta_3 (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_3^2 (\beta_{2x}-1) - 2\Delta_3 (\theta-1)]$ |
| $\hat{\tau}_4^2 = s_y^2\left[\frac{\beta_{1x}S_x^2 + M_d}{\beta_{1x}s_x^2 + M_d}\right]$ J.Subramani* | $\gamma s_y^2 \Delta_4 [\Delta_4 (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_4^2 (\beta_{2x}-1) - 2\Delta_4 (\theta-1)]$ |
| $\hat{\tau}_5^2 = s_y^2\left[\frac{\beta_{2x}S_x^2 + S_x}{\beta_{2x}s_x^2 + S_x}\right]$ J.Subramani* | $\gamma s_y^2 \Delta_5 [\Delta (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_5^2 (\beta_{2x}-1) - 2\Delta_5 (\theta-1)]$ |
| $\hat{\tau}_6^2 = s_y^2\left[\frac{\beta_{2x}S_x^2 + M_d}{\beta_{2x}s_x^2 + M_d}\right]$ J.Subramani* | $\gamma s_y^2\Delta_6 [\Delta_6 (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_6^2 (\beta_{2x}-1) - 2\Delta_6 (\theta-1)]$ |
| $\hat{\tau}_7^2 = s_y^2\left[\frac{\beta_{2x}S_x^2 + \rho}{\beta_{2x}s_x^2 + \rho}\right]$ J.Subramani* | $\gamma s_y^2\Delta_7 [\Delta_7 (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_7^2 (\beta_{2x}-1) - 2\Delta_7 (\theta-1)]$ |
| $\hat{\tau}_8^2 = s_y^2\left[\frac{\beta_{2x}S_x^2 + \beta_{1x}}{\beta_{2x}s_x^2 + \beta_{1x}}\right]$ J.Subramani* | $\gamma s_y^2\Delta_8 [\Delta_8 (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_8^2 (\beta_{2x}-1) - 2\Delta_8 (\theta-1)]$ |
| $\hat{\tau}_9^2 = s_y^2\left[\frac{\rho S_x^2 + \beta_{2x}}{\rho s_x^2 + \beta_{2x}}\right]$ J.Subramani* | $\gamma s_y^2\Delta_9 [\Delta_9 (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_9^2 (\beta_{2x}-1) - 2\Delta_9 (\theta-1)]$ |
| $\hat{\tau}_{10}^2 = s_y^2\left[\frac{\rho S_x^2 + \beta_{1x}}{\rho s_x^2 + \beta_{1x}}\right]$ J.Subramani* | $\gamma s_y^2\Delta_{10} [\Delta_{10} (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_{10}^2 (\beta_{2x}-1) - 2\Delta_{10} (\theta-1)]$ |
| $\hat{\tau}_{11}^2 = s_y^2\left[\frac{\rho S_x^2 + S_x}{\rho s_x^2 + S_x}\right]$ J.Subramani* | $\gamma s_y^2\Delta_{11} [\Delta_{11} (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_{11}^2 (\beta_{2x}-1) - 2\Delta_{11} (\theta-1)]$ |
| $\hat{\tau}_{12}^2 = s_y^2\left[\frac{\rho S_x^2 + M_d}{\rho s_x^2 + M_d}\right]$ J.Subramani* | $\gamma s_y^2\Delta_{12} [\Delta_{12} (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_{12}^2 (\beta_{2x}-1) - 2\Delta_{12} (\theta-1)]$ |
| $\hat{\tau}_{13}^2 = s_y^2\left[\frac{S_x S_x^2 + \beta_{2x}}{S_x s_x^2 + \beta_{2x}}\right]$ J.Subramani* | $\gamma s_y^2\Delta_{13} [\Delta_{13} (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_{13}^2 (\beta_{2x}-1) - 2\Delta_{13} (\theta-1)]$ |
| $\hat{\tau}_{14}^2 = s_y^2\left[\frac{S_x S_x^2 + \beta_{1x}}{S_x s_x^2 + \beta_{1x}}\right]$ J.Subramani* | $\gamma s_y^2\Delta_{14} [\Delta_{14} (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_{14}^2 (\beta_{2x}-1) - 2\Delta_{14} (\theta-1)]$ |
| $\hat{\tau}_{15}^2 = s_y^2\left[\frac{S_x S_x^2 + \rho}{S_x s_x^2 + \rho}\right]$ J.Subramani* | $\gamma s_y^2\Delta_{15} [\Delta_{15} (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_{15}^2 (\beta_{2x}-1) - 2\Delta_{15} (\theta-1)]$ |

| | | |
|---|---|---|
| $\hat{\tau}_{16}^{2} = s_y^2\left[\frac{S_x S_x^2 + M_d}{S_x s_x^2 + M_d}\right]$ J.Subramani* | $\gamma s_y^2 \Delta_{16} [\Delta_{16} (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_{16}^2 (\beta_{2x}-1) - 2\Delta_{16} (\theta-1)]$ |
| $\hat{\tau}_{17}^{2} = s_y^2\left[\frac{M_d S_x^2 + \beta_{2x}}{M_d s_x^2 + \beta_{2x}}\right]$ J.Subramani* | $\gamma s_y^2 \Delta_{17} [\Delta_{17} (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_{17}^2 (\beta_{2x}-1) - 2\Delta_{17} (\theta-1)]$ |
| $\hat{\tau}_{18}^{2} = s_y^2\left[\frac{M_d S_x^2 + S_x}{M_{nd} s_x^2 + S_x}\right]$ J.Subramani* | $\gamma s_y^2 \Delta_{18} [\Delta_{18} (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_{18}^2 (\beta_{2x}-1) - 2\Delta_{18} (\theta-1)]$ |
| $\hat{\tau}_{19}^{2} = s_y^2\left[\frac{M_d S_x^2 + \rho}{M_d s_x^2 + \rho}\right]$ J.Subramani* | $\gamma s_y^2 \Delta_{19} [\Delta_{19} (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_{19}^2 (\beta_{2x}-1) - 2\Delta_{19} (\theta-1)]$ |
| $\hat{\tau}_{20}^{2} = s_y^2\left[\frac{M_d S_x^2 + \beta_{1x}}{M_d s_x^2 + \beta_{1x}}\right]$ J.Subramani* | $\gamma s_y^2 \Delta_{20} [\Delta_{20} (\beta_{2x}-1) - (\theta-1)]$ | $\gamma s_y^4 [(\beta-1) + \Delta_{20}^2 (\beta_{2x}-1) - 2\Delta_{20} (\theta-1)]$ |

**Table 3.1: Population Characteristics and Values of Data 1**

| Characteristics | Values |
|---|---|
| $\bar{X}$ | 11.2646 |
| $\bar{Y}$ | 51.8264 |
| $S_x$ | 8.4563 |
| $S_y$ | 18.3569 |
| $C_x$ | 0.7507 |
| $C_y$ | 0.3542 |
| $M_d$ | 7.5750 |
| $Q_d$ | 5.9125 |
| $Q_1$ | 5.1500 |
| $Q_2$ | 7.5750 |
| $Q_3$ | 16.975 |
| $Q_a$ | 22.125 |
| $Q_r$ | 11.8250 |
| $\rho$ | 0.9413 |
| $\beta_{1x}$ | 1.1 |
| $\beta_{2x}$ | 2.8664 |
| n | 20 |
| N | 80 |
| $\beta$ | 2.2667 |
| $\gamma$ | 0.09470 |
| $\Theta$ | 2.2209 |

For estimating the population variance the first degree of approximation is used, the proposed estimators, bias, constant and mean squared error are given below:

**Table 3.2: Constant of the Existing Estimators and Proposed Estimators**

| Estimators | Constants | Estimators | Constants |
|---|---|---|---|
| $\hat{\tau}_1^2$ | 0.9648 | $\hat{\tau}_{14}^2$ | 0.9982 |
| $\hat{\tau}_2^2$ | 0.9882 | $\hat{\tau}_{15}^2$ | 0.9984 |
| $\hat{\tau}_3^2$ | 0.9035 | $\hat{\tau}_{16}^2$ | 0.9876 |
| $\hat{\tau}_4^2$ | 0.9119 | $\hat{\tau}_{17}^2$ | 0.9947 |
| $\hat{\tau}_5^2$ | 0.9597 | $\hat{\tau}_{18}^2$ | 0.9844 |
| $\hat{\tau}_6^2$ | 0.9639 | $\hat{\tau}_{19}^2$ | 0.9982 |
| $\hat{\tau}_7^2$ | 0.9953 | $\hat{\tau}_{20}^2$ | 0.9979 |
| $\hat{\tau}_8^2$ | 0.9945 | $\hat{\tau}_{N\,2}^{\;1}$ | 0.8410 |
| $\tau_9^2$ | 0.9595 | $\hat{\tau}_{N2}^2$ | 0.8170 |
| $\hat{\tau}_{10}^2$ | 0.9837 | $\hat{\tau}_{N3}^2$ | 0.7333 |
| $\hat{\tau}_{11}^2$ | 0.8877 | $\hat{\tau}_{N4}^2$ | 0.8406 |
| $\hat{\tau}_{12}^2$ | 0.8985 | $\hat{\tau}_{N5}^2$ | 0.7763 |
| $\hat{\tau}_{13}^2$ | 0.9954 | $\hat{\tau}_{N6}^2$ | 0.6939 |

**Proposed Estimators**

Taking motivation from Subramani and Kumarapandiyan, (2012a, 2012b, 2012c), we propose the following estimators

**Table 3.3: Biases of the Existing Estimators and Proposed Estimators**

| Estimators | Biases | Estimators | Biases |
|---|---|---|---|
| $\hat{\tau}_1^{\,2}$ | 93.0301 | $\hat{\tau}_{14}^{\,2}$ | 98.2363 |
| $\hat{\tau}_2^{\,2}$ | 96.6636 | $\hat{\tau}_{15}^{\,2}$ | 98.2679 |
| $\hat{\tau}_3^{\,2}$ | 87.8218 | $\hat{\tau}_{16}^{\,2}$ | 96.5697 |
| $\hat{\tau}_4^{\,2}$ | 85.0561 | $\hat{\tau}_{17}^{\,2}$ | 97.6845 |
| $\hat{\tau}_5^{\,2}$ | 92.2468 | $\hat{\tau}_{18}^{\,2}$ | 96.0691 |
| $\hat{\tau}_6^{\,2}$ | 92.8916 | $\hat{\tau}_{19}^{\,2}$ | 98.2363 |
| $\hat{\tau}_7^{\,2}$ | 97.779 | $\hat{\tau}_{20}^{\,2}$ | 98.189 |
| $\hat{\tau}_8^{\,2}$ | 97.6531 | $\hat{\tau}_{N1}^{\,2}$ | 74.8916 |
| $\hat{\tau}_9^{\,2}$ | 92.2161 | $\hat{\tau}_{N2}^{\,2}$ | 71.5866 |
| $\hat{\tau}_{10}^{\,2}$ | 95.9598 | $\hat{\tau}_{N3}^{\,2}$ | 60.5971 |
| $\hat{\tau}_{11}^{\,2}$ | 81.5194 | $\hat{\tau}_{N4}^{\,2}$ | 74.836 |
| $\hat{\tau}_{12}^{\,2}$ | 83.0891 | $\hat{\tau}_{N5}^{\,2}$ | 66.1386 |
| $\hat{\tau}_{13}^{\,2}$ | 97.7948 | $\hat{\tau}_{N6}^{\,2}$ | 55.7128 |

**Table 3.4: Mean Squared Error of the existing and proposed estimators**

| Estimators | MSE | Estimators | MSE |
|---|---|---|---|
| $\hat{\tau}_1^{\,2}$ | 6941.4748 | $\hat{\tau}_{14}^{\,2}$ | 7408.9281 |
| $\hat{\tau}_2^{\,2}$ | 7272.8307 | $\hat{\tau}_{15}^{\,2}$ | 7411.691 |
| $\hat{\tau}_3^{\,2}$ | 6281.0713 | $\hat{\tau}_{16}^{\,2}$ | 7264.7925 |
| $\hat{\tau}_4^{\,2}$ | 6377.4765 | $\hat{\tau}_{17}^{\,2}$ | 7360.8374 |
| $\hat{\tau}_5^{\,2}$ | 6907.422 | $\hat{\tau}_{18}^{\,2}$ | 7222.1662 |
| $\hat{\tau}_6^{\,2}$ | 6958.7755 | $\hat{\tau}_{19}^{\,2}$ | 7408.9281 |
| $\hat{\tau}_7^{\,2}$ | 7369.3313 | $\hat{\tau}_{20}^{\,2}$ | 74047868 |
| $\hat{\tau}_8^{\,2}$ | 7358.1024 | $\hat{\tau}_{N1}^{\,2}$ | 5733.9738 |
| $\hat{\tau}_9^{\,2}$ | 6904.5232 | $\hat{\tau}_{N2}^{\,2}$ | 5565.5876 |
| $\hat{\tau}_{10}^{\,2}$ | 7212.8965 | $\hat{\tau}_{N3}^{\,2}$ | 5159.0648 |
| $\hat{\tau}_{11}^{\,2}$ | 6127.9373 | $\hat{\tau}_{N4}^{\,2}$ | 5730.6299 |
| $\hat{\tau}_{12}^{\,2}$ | 6231.5274 | $\hat{\tau}_{N5}^{\,2}$ | 5332.492 |
| $\hat{\tau}_{13}^{\,2}$ | 7370.4162 | $\hat{\tau}_{N6}^{\,2}$ | 5064.8053 |

**1st Proposed Estimators**

1st proposed estimator we used linear combination of $M_d$, $Q_1$ and $\rho$.

$$\hat{\tau}_{Nj}^2 = s_y^2\left[\frac{\rho S_x^2 + M_d + Q_1}{\rho s_x^2 + M_d + Q_1}\right]$$

The expression for the Bias of the 1st proposed estimator.

$\text{Bias}(\hat{\tau}_{Nj}^{\,2}) = \varphi\, s^2_y \Delta_2 [\Delta_2 (\beta_{2x} - 1) - (\theta - 1)]$

Similarly the expression for MSE of the 1st proposed estimator.

$\text{MSE}\ (\hat{\tau}_{Nj}^{\,2}) = \varphi\, s^2_y [(\beta_{2x} - 1) - 2\Delta_2 (\theta - 1)]$

Where constant.

$$\Delta_i = \frac{\rho S_{x^2}}{\rho S_{x^2} + M_d + Q_1}$$

**2nd Proposed Estimators**

2nd proposed estimator we used linear combination of $M_d$, $Q_2$ and $\rho$.

$$\hat{\tau}_{Nj}^2 = s_y^2\left[\frac{\rho S_x^2 + M_d + Q_2}{\rho s_x^2 + M_d + Q_2}\right]$$

The expression for the Bias of the 2nd proposed estimator.

$\text{Bias}(\hat{\tau}_{Nj}^{\,2}) = \varphi\, s^2_y\ \Delta_2 [\Delta_2 (\beta_{2x} - 1) - (\theta - 1)]$

Similarly the expression for MSE of the 2nd proposed estimator.

$\text{MSE}(\hat{\tau}_{Nj}^{\,2}) = \varphi\, s^4_y\ [(\beta - 1) + \Delta^2_2 (\beta_{2x} - 1) - 2\Delta_2 (\theta - 1)]$

Where constant.

$$\Delta_2 = \frac{S_{x^2}}{S_{x^2} + M_d + Q_2}$$

**3rd Proposed Estimators**

3rd proposed estimator we used linear combination of $M_d$, $Q_3$ and $\rho$.

$$\hat{\tau}_{Nj}^2 = s_y^2\left[\frac{\rho S_x^2 + M_d + Q_3}{\rho s_x^2 + M_d + Q_3}\right]$$

**Table 3.5: Population Characteristics and Values of Data 2**

| Characteristics | Values |
|---|---|
| $\bar{X}$ | 46.37 |
| $\bar{Y}$ | 37.92 |
| $S_x$ | 25.4 |
| $S_y$ | 14.87 |
| $C_x$ | 0.5478 |
| $C_y$ | 0.3922 |
| $M_d$ | 38.4 |
| $Q_d$ | 6.9 |
| $Q_1$ | 33.9 |
| $Q_2$ | 38.4 |
| $Q_3$ | 47.7 |
| $Q_a$ | 81.6 |
| $Q_r$ | 40.8 |
| $\rho$ | 0.9773 |
| $\beta_{1x}$ | 2.364 |
| $\beta_{2x}$ | 8.269 |
| N | 7 |
| N | 33 |
| $\beta$ | 8.282 |
| $\gamma$ | 0.1126 |
| $\theta$ | 5.51514 |

The expression for the Bias of the 3rd proposed estimator.

$\text{Bias}(\hat{\tau}_{Nj}^2) = \varphi s_y^2 \Delta_3 [\Delta_3 (\beta_{2x} -1) - (\theta - 1)]$

Similarly the expression for MSE of the 3rd proposed estimator.

$\text{MSE}(\hat{\tau}_{Nj}^2) = \varphi s_y^4 [(\beta - 1) + \Delta^3{}_2 (\beta_{2x} - 1) - 2\Delta_3 (\theta - 1)]$

Where constant.

$$\Delta_3 = \frac{S_{x^2}}{S_{x^2} + M_d + Q_3}$$

## 4th Proposed Estimators

4th proposed estimator we used linear combination of $M_d$, $Q_d$ and $\rho$.

$$\hat{\tau}_{Nj}^2 = s_y^2 \left[\frac{\rho S_x^2 + M_d + Q_d}{\rho s_x^2 + M_d + Q_d}\right]$$

The expression for the Bias of the 4th proposed estimator.

$\text{Bias}(\hat{\tau}_{Nj}^2) = \varphi s_y^2 \Delta_4 [\Delta_4 (\beta_{2x} - 1) - (\theta - 1)]$

**Table 3.6: Constant of the Existing Estimators and Proposed Estimators**

| Estimators | Constants | Estimators | Constants |
|---|---|---|---|
| $\hat{\tau}_1^2$ | 0.9946 | $\hat{\tau}_{14}^2$ | 0.9998 |
| $\hat{\tau}_2^2$ | 0.9871 | $\hat{\tau}_{15}^2$ | 0.9999 |
| $\hat{\tau}_3^2$ | 0.9836 | $\hat{\tau}_{16}^2$ | 0.9977 |
| $\hat{\tau}_4^2$ | 0.9754 | $\hat{\tau}_{17}^2$ | 0.9997 |
| $\hat{\tau}_5^2$ | 0.9953 | $\hat{\tau}_{18}^2$ | 0.999 |
| $\hat{\tau}_6^2$ | 0.9929 | $\hat{\tau}_{19}^2$ | 0.9999 |
| $\hat{\tau}_7^2$ | 0.9998 | $\hat{\tau}_{20}^2$ | 0.9999 |
| $\hat{\tau}_8^2$ | 0.9996 | $\hat{\tau}_{N1}^2$ | 0.8971 |
| $\hat{\tau}_9^2$ | 0.987 | $\hat{\tau}_{N2}^2$ | 0.8914 |
| $\hat{\tau}_{10}^2$ | 0.9963 | $\hat{\tau}_{N3}^2$ | 0.8799 |
| $\hat{\tau}_{11}^2$ | 0.9613 | $\hat{\tau}_{N4}^2$ | 0.933 |
| $\hat{\tau}_{12}^2$ | 0.9426 | $\hat{\tau}_{N5}^2$ | 0.8884 |
| $\hat{\tau}_{13}^2$ | 0.9995 | $\hat{\tau}_{N6}^2$ | 0.8401 |

Similarly the expression for MSE of the 4th proposed estimator.

$\text{MSE}(\hat{\tau}_{Nj}^2) = \varphi s_y^2 \Delta_4 [(\beta\text{-}1) + \Delta_4^2 (\beta_{2x}\text{-}1) - 2\Delta_4 (\theta - 1)]$

Where constant.

$$\Delta_4 = \frac{S_{x^2}}{S_{x^2} + M_d + Q_d}$$

## 5th Proposed Estimators

In 5th proposed estimator we used linear combination of $M_d$, $Q_r$ and $\rho$.

$$\hat{\tau}_{Nj}^2 = s_y^2 \left[\frac{\rho S_x^2 + M_d + Q_r}{\rho s_x^2 + M_d + Q_r}\right]$$

The expression for the Bias of the 5th proposed estimator.

$\text{Bias}(\hat{\tau}_{Nj}^2) = \varphi s_y^2 \Delta_5 [\Delta_5 (\beta_{2x} - 1) - (\theta - 1)]$

Similarly the expression for MSE of the 5th proposed estimator.

$\text{MSE}(\hat{\tau}_{Nj}^2) = \varphi s_y^4 [(\beta\text{-}1) + \Delta_5^2 (\beta_{2x} - 1) - 2\Delta_5 (\theta - 1)]$

Where constant.

$$\Delta_5 = \frac{\rho S_{x^2}}{\rho S_{x^2} + M_d + Q_r}$$

**Table 3.7: Biases of the Existing Estimators and Proposed Estimators**

| Estimators | Biases | Estimators | Biases |
|---|---|---|---|
| $\hat{\tau}_1^2$ | 290.8423 | $\hat{\tau}_{14}^2$ | 293.3039 |
| $\hat{\tau}_2^2$ | 287.3093 | $\hat{\tau}_{15}^2$ | 293.3513 |
| $\hat{\tau}_3^2$ | 285.6676 | $\hat{\tau}_{16}^2$ | 292.3086 |
| $\hat{\tau}_4^2$ | 281.8385 | $\hat{\tau}_{17}^2$ | 293.2564 |
| $\hat{\tau}_5^2$ | 291.1713 | $\hat{\tau}_{18}^2$ | 292.9245 |
| $\hat{\tau}_6^2$ | 290.0397 | $\hat{\tau}_{19}^2$ | 293.3513 |
| $\hat{\tau}_7^2$ | 293.3039 | $\hat{\tau}_{20}^2$ | 293.3513 |
| $\hat{\tau}_8^2$ | 293.209 | $\hat{\tau}_{N1}^2$ | 246.5013 |
| $\hat{\tau}_9^2$ | 287.2624 | $\hat{\tau}_{N2}^2$ | 244.0155 |
| $\hat{\tau}_{10}^2$ | 291.646 | $\hat{\tau}_{N3}^2$ | 239.0361 |
| $\hat{\tau}_{11}^2$ | 275.3113 | $\hat{\tau}_{N4}^2$ | 262.4277 |
| $\hat{\tau}_{12}^2$ | 266.7656 | $\hat{\tau}_{N5}^2$ | 242.7119 |
| $\hat{\tau}_{13}^2$ | 293.1616 | $\hat{\tau}_{N6}^2$ | 222.1726 |

**Table 3.8: Mean Squared Error of the existing and proposed estimators**

| Estimators | MSE | Estimators | MSE |
|---|---|---|---|
| $\hat{\tau}_1^2$ | 30231.1766 | $\hat{\tau}_{14}^2$ | 30387.6866 |
| $\hat{\tau}_2^2$ | 30009.2527 | $\hat{\tau}_{15}^2$ | 30390.7176 |
| $\hat{\tau}_3^2$ | 29907.229 | $\hat{\tau}_{16}^2$ | 30324.2201 |
| $\hat{\tau}_4^2$ | 29672.04112 | $\hat{\tau}_{17}^2$ | 30384.6564 |
| $\hat{\tau}_5^2$ | 30252.1192 | $\hat{\tau}_{18}^2$ | 30363.4673 |
| $\hat{\tau}_6^2$ | 30180.4793 | $\hat{\tau}_{19}^2$ | 30390.7176 |
| $\hat{\tau}_7^2$ | 30387.6866 | $\hat{\tau}_{20}^2$ | 30390.7176 |
| $\hat{\tau}_8^2$ | 30381.627 | $\hat{\tau}_{N1}^2$ | 27697.9431 |
| $\hat{\tau}_9^2$ | 30006.3242 | $\hat{\tau}_{N2}^2$ | 27573.1733 |
| $\hat{\tau}_{10}^2$ | 30282.1025 | $\hat{\tau}_{N3}^2$ | 27328.2792 |
| $\hat{\tau}_{11}^2$ | 29280.2158 | $\hat{\tau}_{N4}^2$ | 28541.0513 |
| $\hat{\tau}_{12}^2$ | 28785.1065 | $\hat{\tau}_{N5}^2$ | 27508.3284 |
| $\hat{\tau}_{13}^2$ | 30378.5984 | $\hat{\tau}_{N6}^2$ | 26568.5882 |

### 6th Proposed Estimators

In 6th proposed estimator we used linear combination of $M_d$, $Q_a$ and ρ.

$$\hat{\tau}_{Nj}^2 = s_y^2\left[\frac{\rho S_x^2 + M_d + Q_a}{\rho s_x^2 + M_d + Q_a}\right]$$

The expression for the Bias of the 6th proposed estimator.

$\text{Bias}(\hat{\tau}_{Nj}^2) = \varphi s_y^2 \Delta_5 [\Delta_5 (\beta_{2x} - 1) - (\theta - 1)]$

Similarly the expression for MSE of the 6th proposed estimator.

$\text{MSE}(\hat{\tau}_{Nj}^2) = \varphi s_y^4 [(\beta-1) + \Delta_6^2 (\beta_{2x} - 1) - 2\Delta_6 (\theta - 1)]$

Where constant.

$$\Delta_6 = \frac{\rho s_x^2}{\rho s_x^2 + M_d + Q_r}$$

Note that, The purpose of adding Md and $Q_1$, $Q_2$, $Q_3$, $Q_d$, $Q_r$, $Q_a$ in is to minimize $\Delta_1, \ldots, \Delta_6$. The minimization in $\Delta_1, \ldots, \Delta_6$ will reduce mean square error and hence we get better results.

### Empiricalstudy

**Population-1**

In the first population, the mean of the auxiliary variable $\bar{X}$ = 11.2646 and the standard deviation $S_x$= 8.4563 respectively.

Auxiliary variable and study variable are highly correlated with ρ=0.9413. Both the variable contains 80 units. We calculated X elements on the auxiliary characteristic and Y elements on the study characteristics. Another fact of interest is that in our population the efficiency gain achieved by the proposed estimators. The Population is taken from the Murthy (1967, Page 228). Descriptive statistics, constants, bias, mean squared error are given below:

The characteristics, constants, bias and mean squared error of the proposed and existing estimators are given in table 4.1, 4.2, 4.3, 4.4 respectively. We use the linear combination of measures of location for numerical illustrations. However these all existing

ratio variance estimators are biased but have minimum values of bias and mean squared error as compared to classical ratio estimators.
New modified variance ratio estimators introduced by using linear combination of measures of location shows better results than the existing modified variance ratio estimators.

**Population-2**
In the 2ndpopulation, the mean of the auxiliary variable $\bar{X}$= 46.37 and the standard deviation $S_x$ = 25.4 respectively.

Auxiliary variable and study variable are highly correlated with $\rho$=0.9773. Both the variable contains 33 units. We calculated X elementson the auxiliary characteristic and Y elements on the study characteristics. Another fact of interest is that in our population the efficiency gain achieved by the proposed estimators. The Population Data 2 is taken from Government of Pakistan Statistics Division Federal Bureau of Statistics(Economic Wing)Islamabad (AREA & PRODUCTION OF FOOD CROPS IN PUNJAB, Area in "000" Hectares, Production in "000" Tones, Barley) from 1981-2014. Descriptive statistics, constants, bias, mean squared error are given below:

The characteristics, constants, bias and mean squared error of the proposed and existing estimators are given in table 4.5, 4.6, 4.7, 4.8 respectively. We use the linear combination of measures of location for numerical illustrations. However these all existing ratio variance estimators are biased but have minimum values of bias and mean squared error as compared to classical ratio estimators till date no attempt has been made to utilized the linear combination of measures of location.

New modified variance ratio estimators introduced by using linear combination of measures of location shows better results than the existing modified variance ratio estimators.

Note that, on replacing the unknown population quantities in the optimum values of constants of an estimator of interest with their respective consistent estimators based on the same sample, the efficiency of the estimator of interest remains the same, up to first order of approximation.

**Conclusion**
This Research proposed ratio type variance estimator by using a known linear combination of median quartile and correlation coefficient of an auxiliary variable. The bias, mean squared error of the planned estimator were obtained and compared with the typical ratio kind and obtainable modified ratio kind variance estimators. Further the conditions for which the planned estimator is more capable than the conventional and accessible estimators were derived. The performance of the plannedestimator was experienced using five knownpopulations. Results explain that the bias, mean squared error of the planned estimator arelesser than the biased, mean squared errors of the conventional and existing estimators for the known populations measured. Based on results, the planned modified ratio type variance estimator may be favored over conventional ratio kind and obtainable modified ratio type variance estimators for the use in realistic applications.

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