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Improved Estimator of Population Variance using Measure of Dispersion of Auxiliary Variable

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Abstract

This research study is designed to obtain a more precise class of estimators of a population variance by taking advantage of relation between auxiliary variable and study variable. Here a class of new modified ratio type estimators of population variance by using coefficient of variation (CV), standard deviation, mean and median of auxiliary variable. Further empirical study is made to compare bias and mean square error (MSE) of proposed estimators with the existing estimators. Expressions for bias and MSE are obtained. Few secondary data sets are used to check the efficiency of proposed estimators of population variance.

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Introduction

In our everyday life variations are available all over the place. It is the idea of law that people or no two things are precisely same. For example, an agriculturist needs a sufficient comprehension of the varieties in climatic factors particularly from place to place (or time to time) to have the capacity to anticipate when, how and where to plant his yield. For consistent learning of the level of varieties in individuals' response a maker need to decrease or increase cost of his item, or make strides the nature of his item. A doctor needs a full comprehension of varieties in the body temperature, level of human circulatory strain and heartbeat rate for full remedy. In this article we estimate one of the measure of variation which is known as variance.

The following Notations are used throughout this paper.

"N" : Size of Population

"n" : Size of Sample

γ : $1/n$

Y : Variate of interest

X : Supplementary variate

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$\bar{X}$: A.M of Supplementary variate
$\bar{Y}$: A.M of Variate of interest
$\bar{y}$: Sample A.M of Variate of interest
$\bar{x}$: Sample A.M of Supplementary variate
ρ : Correlation coefficient
S_y^2 : Variance of Variate of interest
S_x^2 : Variance of Supplementary variate
s_y^2 : Sample Variance of Study variable
s_x^2 : Sample Variance of Auxiliary variable
C_y : CV of Study variate
C_x : CV of Supplementary variate
Md : Population Median of Auxiliary variable
B(.) : Bias of the estimator
MSE(.) : Mean squared error of the estimato
β_1= Coefficient of skewness
β_2 = Coefficient of Kurtosis

$\hat{S}^2_{KCi}$: Existing improved ratio type variance estimators of S^2_y by Kadilar &Cingi, (2006a)

$\hat{S}^2_{JG}$: Existing improved ratio type variance estimator of S^2_y bySubramani and Kumarapandiyan (2012)

$\hat{S}^2_{NKi}$: Proposed ratio type variance estimators of S^2_y

KC(2006)and SK (2012) proposed a class ofratio type variance estimators for the population variance S^2_y. It is assumed that variance of supplementary variable (S^2_x)is known. Family members of KC(2006) and SK (2012) utilizing supplementary information with their theoretical properties such as bias and mean squared error are given below.

Where $\beta_{2(y)} = \frac{\mu_{40}}{\mu_{20}^2}$, $\beta_{2(x)} = \frac{\mu_{04}}{\mu_{02}^2}$, $\lambda_{22} = \frac{\mu_{22}}{\mu_{20}\mu_{02}}$

and $\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}(y-\bar{Y})^r(x_i-\bar{X})^s$

Table 1: Reviewed variance estimators with their theoretical properties

Estimators	Bias (.)	Mean Squared error MSE (.)
$\hat{S}^2_{KC1} = s_y^2\left[\frac{S_x^2+C_x}{s_x^2+C_x}\right]$ Kadilar and Cingi (2006)	$\gamma S_y^2 A_1\left[A_1\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right]$	$\gamma S_y^4\left[\begin{matrix}\left(\beta_{2(y)}-1\right)+A_1^2\left(\beta_{2(x)}-1\right)\\-2A_1\left(\lambda_{22}-1\right)\end{matrix}\right]$
$\hat{S}^2_{KC2} = s_y^2\left[\frac{S_x^2+\beta_{2(x)}}{s_x^2+\beta_{2(x)}}\right]$ Kadilar and Cingi (2006)	$\gamma S_y^2 A_2\left[A_2\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right]$	$\gamma S_y^4\left[\begin{matrix}\left(\beta_{2(y)}-1\right)+A_2^2\left(\beta_{2(x)}-1\right)\\-2A_2\left(\lambda_{22}-1\right)\end{matrix}\right]$
$\hat{S}^2_{KC3} = s_y^2\left[\frac{S_x^2\beta_{2(x)}+C_x}{s_x^2\beta_{2(x)}+C_x}\right]$ Kadilar and Cingi (2006)	$\gamma S_y^2 A_3\left[A_3\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right]$	$\gamma S_y^4\left[\begin{matrix}\left(\beta_{2(y)}-1\right)+A_3^2\left(\beta_{2(x)}-1\right)\\-2A_3\left(\lambda_{22}-1\right)\end{matrix}\right]$
$\hat{S}^2_{KC4} = s_y^2\left[\frac{S_x^2C_x+\beta_{2(x)}}{s_x^2C_x+\beta_{2(x)}}\right]$ Kadilar and Cingi (2006)	$\gamma S_y^2 A_4\left[A_4\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right]$	$\gamma S_y^4\left[\begin{matrix}\left(\beta_{2(y)}-1\right)+A_4^2\left(\beta_{2(x)}-1\right)\\-2A_4\left(\lambda_{22}-1\right)\end{matrix}\right]$
$\hat{S}^2_{SK} = s_y^2\left[\frac{S_x^2+Md}{s_x^2+Md}\right]$	$\gamma S_y^2 A_{SK}\left[A_{SK}\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right]$	$\gamma S_y^4\left[\begin{matrix}\left(\beta_{2(y)}-1\right)+A_{SK}^2\left(\beta_{2(x)}-1\right)\\-2A_{SK}\left(\lambda_{22}-1\right)\end{matrix}\right]$

$$A_I = \frac{S_X^2}{S_X^2 + C_X}, \quad A_2 = \frac{S_X^2}{S_X^2 + \beta_X}, \quad A_3 = \frac{S_X^2 \beta_{2(x)}}{S_X^2 \beta_{2(x)} + C_X}$$

$$, A_4 = \frac{S_X^2 C_{2(x)}}{S_X^2 C_{2(x)} + \beta_{2(x)}} \text{ and } A_{SK} = \frac{S_x^2}{S_x^2 + Md}$$

The B(.) and MSE(.) of existing estimators up to first order of approximation are as follows:

$$B\left(\hat{S}_{SK}^2\right) = \gamma S_y^2 A_{SK}\left[A_{SK}\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \quad ....(1)$$

$$MSE\left(\hat{S}_{SK}^2\right) = \gamma S_y^4 \begin{bmatrix} \left(\beta_{2(y)} - 1\right) + A_{SK}^2\left(\beta_{2(x)} - 1\right) \\ - 2A_{SK}\left(\lambda_{22} - 1\right) \end{bmatrix} \quad ....(2)$$

Where $A_{SK} = \frac{S_x^2}{S_x^2 + Md}$

Table 2: Decriptiveof the considered Populations

Characteristics	Pop- 1	Pop- 2	Pop- 3	Pop- 4	Pop- 5
N	22	49	36	103	103
n	5	20	12	40	40
$\bar{Y}$	22.5	116.1633	897.075	626.2123	62.6212
$\bar{X}$	1467.5	98.6765	22.97778	557.1909	556.5541
ρ	0.9022	0.6904	0.9413	0.9936	0.7298
S_y	32.8	98.8286	1382.504371	913.5498	91.3549
C_y	1.45777778	0.8508	1.541124623	1.4588	1.4588
S_x	2503.2	102.9709	32.10377914	818.1117	610.1643
C_x	1.705758092	1.0435	1.397166403	1.4683	1.0963
$\beta_{2(x)}$	13.2	5.9878	3.525665114	37.3216	17.8738
$\beta_{2(x)}$	5.57	4.9245	4.581638511	37.1279	37.1279
λ_{22}	7.71	4.6977	2.657582	37.2055	17.2220
Md	534.5	64	11.7	308.05	373.82
A_{KC1}	0.999999728	0.999901594	0.998646222	0.999997806	0.99999706
A_{KC2}	0.999997893	0.999435592	0.996590854	0.999944242	0.99995199
A_{KC3}	0.999999979	0.999983564	0.999615649	0.999999941	0.99999984
A_{KC4}	0.999998765	0.999459108	0.997557590	0.999962024	0.999995621
A_{SK}	0.999914706	0.994000191	0.988775392	0.999539959	0.99899693
A_{NK1}	0.999319033	0.989967735	0.958294717	0.998208473	0.99820649
A_{NK2}	0.999600671	0.990382107	0.969791966	0.998779148	0.99836382
A_{NK3}	0.999854517	0.993740833	0.984386958	0.999324668	0.99890044

The purpose ofabove mentionedratioestimators is to reduce the varianceof estimates when there exist a positive relation between X and Y. The problem of estimating S^2_y has been extensively discussed by many authors namelyIsaki[5], Singh *et al.*[13], Cebrian and Garcia[3], Ahmad *et al.*[1], Singh & Singh[11], KC[7], Singh & Singh[11], Shabbir & Gupta[10], KC[6], Gupta and Shabbir[4], Singh and Solanki[12], SK[17,18], Khan and Shabbir[8] and Yadav *et al.*[20,21,22] etc.

The improved ratio type variance estimators discussed above are biased but have minimum mean square errors compared to the old ratio type variance estimator. The list of estimators given in table 1 uses the known values of the parameters like S^2_χ, C_χ, β_2, median and their linear combinations. The materials of the present study are arranged as given below. The proposed estimators with known population C_χ, S_χ, mean, median are presented in section 2 and the condition in which the proposed estimator performs better than the existing estimators are derived in section 3. The performances of the proposed and the existing estimators are measured for certain natural populations in section 4 and conclusion is presented in section 5.

Proposed Estimators

The accurateness of the estimator may be increased by using the supporting evidence in ratio type variance estimator. Estimators depend on the population characteristics in applied field. We use ratio estimator to estimate the values of the population variance by using different characteristics like kurtosis, skewness, mean, median, quartiles. deciles , percentiles and coefficient of variation etc. Supporting evidences are utilizedin sampling survey to improve the estimate of S^2_y. In current section, we have proposed a class of ratio type variance estimator utilizing different parameters of auxiliary variate.

The new and improved class of ratio type variance estimators for population variance S^2_y is defined as

Table 3: B(.) of the Reviewed and New estimators

Estimator	Bias (.)				
	Pop- 1	Pop- 2	Pop- 3	Pop- 4	Pop- 5
$\hat{S}^2_{KC1}$ Kadilar and Cingi (2006)	1181.271284	629.7246080	137534.2245	2420.681024	135.982703
$\hat{S}^2_{KC2}$ Kadilar and Cingi (2006)	1181.264302	629.3152980	136427.1447	2379.9609	135.817938
$\hat{S}^2_{KC3}$ Kadilar and Cingi (2006)	1181.272242	629.9758923	138057.5651	2422.304133	135.992868
$\hat{S}^2_{KC4}$ Kadilar and Cingi (2006)	1181.267619	628.3687034	136947.433	2393.47909	135.833355
$\hat{S}^2_{SK}$ Subramaniand Kumarapandiyan (2012)	1180.947682	611.7195019	132248.5488	2072.764148	132.329179
$\hat{S}^2_{NK1}$ Proposed 1	1178.681558	599.5141042	116421.4024	1062.775604	129.446672
$\hat{S}^2_{KC2}$ Proposed 2	1179.752763	600.7646676	122303.5696	1495.327656	130.020039
$\hat{S}^2_{KC3}$ Proposed 3	1180.718622	610.9320957	129923.7838	1909.274388	131.977069

$$\hat{S}^2_{NK1} = s_y^2\left[\frac{S_x^2 + C_x S_x}{s_x^2 + C_x S_x}\right] \quad \hat{S}^2_{NK2} = s_y^2\left[\frac{S_x^2 + C_x \bar{X}}{s_x^2 + C_x \bar{X}}\right]$$

$$\hat{S}^2_{NK3} = s_y^2\left[\frac{S_x^2 + C_x Md}{s_x^2 + C_x Md}\right]$$

The theoretical properties of after some simple algebra are derived as given below:

$$Bias\left(\hat{S}^2_{NKi}\right) = \gamma S_y^2 A_{NKi}\left[A_{NKi}\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \quad ....(3)$$

$$MSE\left(\hat{S}^2_{NKi}\right) = S_y^4 \gamma\begin{pmatrix}\left(\beta_{2(y)} - 1\right) + A^2_{NKi}\left(\beta_{2(x)} - 1\right) \\ -2A_{NKi}\left(\lambda_{22} - 1\right)\end{pmatrix} \quad ....(4)$$

Theoretical Comparisons of Proposed Estimators

As mentioned the B(.) and MSE(.) of KC (2006) variance estimators are given below:

$$B\left(\hat{S}^2_{KCi}\right) = \gamma S_y^2 A_{KCi}\left[A_{KCi}\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \quad ....(5)$$

$$MSE\left(\hat{S}^2_{KCi}\right) = \gamma S_y^4\left[\begin{matrix}\left(\beta_{2(y)} - 1\right) + A^2_{KCi}\left(\beta_{2(x)} - 1\right) \\ -2A_{KCi}\left(\lambda_{22} - 1\right)\end{matrix}\right] \quad ....(6)$$

As mentioned the B(.) and MSE(.) of SK (2012) variance estimator are given below:

$$B\left(\hat{S}^2_{SK}\right) = \gamma S_y^2 A_{SK}\left[A_{SK}\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \quad ....(7)$$

$$MSE\left(\hat{S}^2_{SK}\right) = \gamma S_y^4\left[\begin{matrix}\left(\beta_{2(y)} - 1\right) + A^2_{SK}\left(\beta_{2(x)} - 1\right) \\ -2A_{SK}\left(\lambda_{22} - 1\right)\end{matrix}\right] \quad ....(8)$$

The B(.) and MSE(.) of proposed class ($\hat{S}^2_{NKi}$) are derived as given below.

$$Bias\left(\hat{S}^2_{NKi}\right) = \gamma S_y^2 A_{NKi}\left[A_{NKi}\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \quad ....(9)$$

$$MSE\left(\hat{S}^2_{NKi}\right) = S_y^4 \gamma\begin{pmatrix}\left(\beta_{2(y)} - 1\right) + A^2_{NKi}\left(\beta_{2(x)} - 1\right) \\ -2A_{NKi}\left(\lambda_{22} - 1\right)\end{pmatrix} \quad ....(10)$$

Table 4: MSE(.) of the Reviewed and New estimators

Estimator	MSE(.)				
	Pop- 1	Pop- 2	Pop- 3	Pop- 4	Pop- 5
$\hat{S}^2_{KC1}$ Kadilar and Cingi (2006)	775478.5508	7234105.562	84929178.23	670384402.9	35796604.9
$\hat{S}^2_{KC2}$ Kadilar and Cingi (2006)	775473.8884	7228377.799	84821296.94	670169790.4	35796502.6
$\hat{S}^2_{KC3}$ Kadilar and Cingi (2006)	775479.1903	7235114.148	84980286.58	670393032.1	35796611.2
$\hat{S}^2_{KC4}$ Kadilar and Cingi(2006)	775476.1037	7228666.588	84871957.66	670240637.1	35796512.2
$\hat{S}^2_{SK}$ Subramaniand Kumarapandiyan (2012)	775262.4693	7162332.886	84417013.86	668667060.7	35794364.2
$\hat{S}^2_{NK1}$ Proposed 1	773749.7250	7114243.297	82930041.76	665179596.5	35792634.9
$\hat{S}^2_{KC2}$ Proposed 2	774464.7103	7119149.271	83474143.18	666399723.9	35792976.2
$\hat{S}^2_{KC3}$ Proposed 3	775109.5267	7159216.595	84194124.07	667951178.5	35794151.1

From the expression given in (6) and (8) we have derived the condition for which the Subramani and Kumarapandiyan (2012) estimator $\hat{S}^2_{NKi}$ is more efficient than Kadilar and Cingi (2006) estimators and it is given below

$$MSE(\hat{S}^2_{SK}) < MSE(\hat{S}^2_{KCi})$$

if $\lambda_{22} > 1 + \frac{(A_{SK} + A_{KCi})(\beta_{2(x)} - 1)}{2}$

From the (8) and (10) we have obtained the theoretical condition for which the proposed estimators $\hat{S}^2_{NKi}$ are more efficient than SK (2012) estimator $\hat{S}^2_{NKi}$ and it is given below

$$MSE(\hat{S}^2_{NKi}) < MSE(\hat{S}^2_{SK})$$

If $\lambda_{22} > 1 + \frac{(A_{NKi} + A_{SK})(\beta_{2(x)} - 1)}{2}$

Numerical Study

The performance of the proposed ratio type variance estimators are evaluated with that of existing modified ratio type estimators listed in Table 1 for certain natural population. The population 1 has taken from Singh and Chaudhary (1986, page 141), population 2 has taken from Cochran (1977, page 151), population 3 is real data set taken from Bureau of statistics. The data is about area and production of sugarcane in the districts of Punjab. Where Y= Production of sugar cane in Punjab in 2008-2009 and X= Area of sugar cane in Punjab in 2007-2008, Population 4 and 5 are real data set taken from the Report on Waste 2004 drew up by the Italian bureau for the environmental-protection.

2003X=Number of inhabitants in 2003. The population parameters and the constants computed from the above populations are given below

The B(.) and MSE(.) of the reviewed and proposed improved ratio type variance estimators are given in the following Tables:

Numerical results of Table 3, shows that the B(.) of the new class of variance estimators is less as compare to the B(.) of the reviewed estimators. Similarly,numerical calculations of Table 4, shows that the MSE(.) of the new class of variance estimators is less as compare to the B(.) of the reviewed estimators.

Conclusions

In this Study we have defined improved some new variance estimators by utilizing coefficient of variation, standard deviation, mean and M_d of supplementary variate. The B(.) and MSE(.) of the new variance estimators are obtained and compared with that of existing improved ratio type variance estimators. Further we have derived some theoretical conditions for which $\hat{S}^2_{NKi}$ estimators are performing much better as compare to the reviewed estimators. So on behalf of the results of population 1, 2, 3, 4 and 5 we claim that the proposed-ratio-type variance estimators are competent as compare to existing ratio type variance estimators.

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