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Some New Non-Travelling Wave Solutions of the Fisher Equation with Nonlinear Auxiliary Equation

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Abstract

We have generated many new non-travelling wave solutions by executing the new extended generalized and improved (G'/G)-Expansion Method. Here the nonlinear ordinary differential equation with many new and real parameters has been used as an auxiliary equation. We have investigated the Fisher equation to show the advantages and effectiveness of this method. The obtained non-travelling solutions are expressed through the hyperbolic functions, trigonometric functions and rational functional forms. Results showing that the method is concise, direct and highly effective to study nonlinear evolution equations those are in mathematical physics and engineering.

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Introduction

NLEE is one of the most powerful and important model equations among all equations in nonlinear sciences and it plays a dynamic role in the field of scientific work or engineering sciences. Those reveal a lot of physical evidence which help to understand better in real world problems. That is why the explicit solutions of NLEEs play significant role in the study of physical phenomena and remains a crucial field for researchers in the ongoing investigation. For the past few decades and so on, a vast research has been carried out to find explicit solutions of NLEEs, and for that substantial work are being made by mathematicians and scientists and have developed effective and convincing methods such as the Hirota's bilinear transformation method,[1] the tanh-coth method,[2,3] the Exp-function method,[4,5] the F-expansion method,[6] the Jacobi elliptic function method,[7] the Weierstrass elliptic function method, the homogeneous balance method,[8] the

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sine –cosine method,[10] the homotopy perturbation method,[11] the direct algebraic method,[12] the Backlund transformation method,[13] the Cole-Hopf transformation method,[14] and others.[15,16]

In 2008, Wang *et. al.*,[17] presented a method called the basic (*G'/G*) - expansion method for the traveling wave solutions of NLEEs. This (*G'/G*) method shows that it is one of the most powerful and effective method to solve NLEEs since it gives a clear and short to the point results in terms of hyperbolic functions, trigonometric functions and rational functional form that is why scientists have carried out a lot of researches to construct traveling wave solutions via this method such as Zayed *et. al.*,[18] Feng *et. al.*,[19] Naher and Abdullah[20] and so on.

Further research of (*G'/G*) method has been continued by a group of researchers to show the possible productivity of the application. For example- Zhang *et. al.*,[21] expanded the (*G'/G*) - expansion method and named as the improved (*G'/G*) method. Vital role of analytical solutions many scientists are continuing their research using aforementioned method for NLPDEs.[22-25] In the meanwhile, Naher and Abdullah demonstrated a new method that is the new approach of the (*G'/G*) method and the new approach of the generalized (*G'/G*) method where nonlinear ODE was used as auxiliary equation and the resulted travelling wave solutions of this method quite different. Still research is carried out using the (*G'/G*) method to generate many new travelling wave solutions of NLEEs.

In this paper, we have executed the new extended generalized and improved (*G'/G*) - expansion method Naher and Abdullah,[27] to illustrate the effectiveness of this method. To do this we have taken the Fisher Equation. As a result many new and more general non-travelling wave solutions are obtained. The rest of the paper is organized as follows: in section 2 we have given the description of the new extension of the generalized and improved (*G'/G*) - expansion method, in section 3 we used the method on Fisher Equation to obtain solutions, in section 4 some discussions are given, in section 5 conclusions are presented.

Algorithms of New Extended Generalized and Improved (G'/G) Method

Recently a new algorithm has been introduced called the new extension of the generalized and improved (*G'/G*) - expansion method for NLEEs. So to demonstrate this method, firstly a NLPDE is taken with two real independent variables x and i.e.

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}...) = 0 \quad ....(2.1)$$

where P is the polynomial and here $u = u(x,t)$ is an unknown function. In the polynomial P contains, different partial derivatives of the function u itself wherein involves the highest order derivatives and the highest nonlinear terms. Now the prime process of the method is being discussed in steps below:

Step1

Let us consider

$$u(x,t) = u(\xi),\ \xi = x \pm Wt \quad ....(2.2)$$

where the constant term W is known as the speed of wave, is substituted in eq (2.1), which allows a PDE to convert an ODE with respect to ξ :

$$Q(u, u', u'', u'''...) = 0, \quad ....(2.3)$$

Step 2

Eq. (2.3) is being integrated term by term and if needed it can be integrated more than once and the integral constants may be set to zero to make it easy to solve. Now the travelling wave solution of Eq. (2.3) can be represented as.

$$u(\xi) = \sum_{j=-N}^{N} a_j \left(d+H\right)^j + \sum_{j=1}^{N} \frac{b_j}{\left(d+H\right)^j} \quad ...(2.4)$$

where a_N, a_{-N}, or b_N can be zero but all cannot be zero at the same time, $a_j (j = 0, \pm1, \pm2,... \pm N)$, $b_j (j = 1, 2, 3,...)$, *d* is the arbitrary constant to be determined later and $H(\xi)$ is

$$H(\xi) = (G'/G) \quad ....(2.5)$$

where $G = G(\xi)$ satisfies the nonlinear ordinary differential equation (ODE) i.e.

$$\lambda GG'' - \mu GG' - \delta(G')^2 - \beta(G)^2 = 0 \quad ....(2.6)$$

where $G'' = d^2G/d\xi^2$, $G' = dG/d\xi$ and λ, μ, δ & β are the real parameters.

Step 3

We have settled the positive integer N by taking into account the homogeneous balance between the highest order derivative and the highest nonlinear terms in Eq. (2.3). Then the value of N is substituted in Eq. (2.4)

Step 4

We substitute the Eq. (2.4) along with Eq. (2.5) into the Eq. (2.3) and then a set of polynomials and its derivatives are obtained. From the resulted polynomials we equate all the coefficients of $(d + H)^j$, ($j = 0, \pm1, \pm2, ... \pm N$) and $(d + H)^j$, ($j = 1, 2, 3, ...N$) to zero and consider as a system of algebraic equations which can solved to determine unknown parameters a_j ($j = 0, \pm1, \pm2, ... \pm N$), b_j ($j = 1, 2, 3, ...N$), d and W. Consequently, we obtain the exact new non travelling wave solutions of Eq. (2.1).

Step 5

With the general solution of Eq. (2.6), the solutions for Eq. (2.5) are obtained as:

Family 1

When $\mu \neq 0$, $\Psi = \lambda - \delta$ and $\Omega = \mu^2 + 4\beta(\lambda - \delta) > 0$,

$$H(\xi) = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{\Omega}}{2\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right) + C_2\sinh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)} \quad ....(2.7)$$

Family 2

When $\mu \neq 0$, $\Psi = \lambda - \delta$ and $\Omega = \mu^2 + 4\beta(\lambda - \delta) < 0$,

$$H(\xi) = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{\sqrt{-\Omega}}{2\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right) + C_2\sin\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)} \quad ....(2.8)$$

Family 3

When $\mu \neq 0$, $\Psi = \lambda - \delta$ and $\Omega = \mu^2 + 4\beta(\lambda - \delta) = 0$,

$$H(\xi) = \left(\frac{G'}{G}\right) = \frac{\mu}{2\Psi} + \frac{C_2}{C_1 + C_2\xi} \quad ....(2.9)$$

Family 4

When $\mu \neq 0$, $\Psi = \lambda - \delta$ and $\Delta = \Psi\beta > 0$

$$H(\xi) = \left(\frac{G'}{G}\right) = \frac{\sqrt{\Delta}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right) + C_2\sinh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)} \quad ....(2.10)$$

Family 5

When $\mu \neq 0$, $\Psi = \lambda - \delta$ and $\Delta = \Psi\beta < 0$

$$H(\xi) = \left(\frac{G'}{G}\right) = \frac{\sqrt{-\Delta}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right) + C_2\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right) + C_2\sin\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)} \quad ....(2.11)$$

Application of the Method

Let us consider the equation to investigate and to construct new non-travelling wave solutions by using the new extended generalized and improved (G'/G) expansion method.

The Fisher equation

$$u_t - u_{xx} - u(1-u) = 0 \quad ....(3.1)$$

By the wave transformation Eq. (2.2), the Eq. (3.1) transforms into the following NLODE:

$$u'' + Wu' + u - u^2 = 0 \quad ....(3.2)$$

Now by considering the homogeneous balance between the nonlinear term u^2 and the highest order derivative u'' in Eq. (3.2), we have the value for N, i.e. $N = 2$. Therefore the solution of Eq. (3.2) can be mentioned as:

$$u(\xi) = a_0 + a_1(d + H) + a_2(d + H)^2 + (a_{-1} + b_1)(d + H)^{-1} + (a_{-2} + b_2)(d + H)^{-2} \quad ...(3.3)$$

where a_{-2}, a_{-1}, a_0, a_1, a_2, b_1, b_2 and d are constants to be determined.

Substituting Eq (3.3) along with Eq (2.5) and (2.6) into Eq (3.2) and by simplifying it transforms into polynomials in $(d + H)^j$ ($j = 0, \pm1, \pm2$) and $(d + H)^{-j}$ ($j = 1,2,3,...$). By collecting the resulted polynomials, yields a set of simultaneous algebraic equations for a_{-2}, a_{-1}, a_0, a_1, a_2, b_1, b_2, d and W. After solving the system of algebraic equations with the aid of Maple, we have obtained the following sets result for non-travelling waves.

Results of Non-travelling waves

Set 1

$$\lambda=\lambda,\ \mu=-2d\Psi,\ \delta=\delta,\ \beta=-\frac{1}{4\Psi}\left(\lambda^2+4d^2\Psi^2\right),\ W=0,\ d=d,\ a_{-2}=-\frac{1}{8\Psi^2}\left(-3\lambda^2+4b_2\Psi^2\right),$$
$$a_{-1}=-b_1,\ a_0=\frac{3}{2},\ a_1=0,\ a_2=0,\ b_1=b_1,\ b_2=b_2; \tag{3.1.1}$$

where, $\Psi=\lambda-\delta$

Set 2

$$\lambda=\lambda,\ \mu=-2d\Psi,\ \delta=\delta,\ \beta=-\frac{1}{4\Psi}\left(-\lambda^2+4d^2\Psi^2\right),\ W=0,\ d=d,\ a_{-2}=-\frac{1}{8\Psi^2}\left(-3\lambda^2+4b_2\Psi^2\right),$$
$$a_{-1}=-b_1,\ a_0=-\frac{1}{2},\ a_1=0,\ a_2=0,\ b_1=b_1,\ b_2=b_2; \tag{3.1.2}$$

where, $\Psi=\lambda-\delta$

Set 3

$$\lambda=\lambda,\ \mu=\mu,\ \delta=-\frac{1}{4\beta}\left(\lambda^2-\mu^2-4\lambda\beta\right),\ \beta=\beta,\ W=0,\ d=d,\ a_{-2}=\frac{1}{8\lambda^2\beta^2}\left(-24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2\right.$$
$$\left.-96\mu d\beta^3-8\lambda^2b_2\beta^2+48\beta^4+3\lambda^4d^4-6\lambda^2\mu^2d^4+3d^4\mu^4+24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right),a_{-1}=-\frac{1}{4\lambda^2\beta^2}$$
$$\left(36\mu^2\beta^2d-12\lambda^2\beta^2d+18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d-6\mu^2d^3\lambda^2+3\mu^4d^3+4\lambda^2b_1\beta^2\right),$$
$$a_0=\frac{1}{8\lambda^2\beta^2}\left(3\lambda^4d^2-6\lambda^2\mu^2d^2+12\lambda^2\mu d\beta-4\lambda^2\beta^2+3\mu^4d^2-12\mu^3d\beta+12\beta^2\mu^2\right),a_1=0,\ a_2=0,$$
$$b_1=b_1,b_2=b_2; \tag{3.1.3}$$

Set 4

$$\lambda=\lambda,\ \mu=\mu,\ \delta=\frac{1}{4\beta}\left(\lambda^2+\mu^2+4\lambda\beta\right),\ \beta=\beta,\ W=0,\ d=d,\ a_{-2}=\frac{1}{8\lambda^2\beta^2}\left(24\lambda^2\beta^2d^2+72\mu^2d^2\beta^2\right.$$
$$\left.-96\mu d\beta^3-8\lambda^2b_2\beta^2+48\beta^4+3\lambda^4d^4+6\lambda^2\mu^2d^4+3d^4\mu^4-24\lambda^2\mu\beta d^3-24\mu^3\beta d^3\right),\ a_{-1}=-\frac{1}{4\lambda^2\beta^2}$$
$$\left(36\mu^2\beta^2d+12\lambda^2\beta^2d-18\lambda^2d^2\beta\mu-18\mu^3\beta d^2-24\mu\beta^3+3\lambda^4d^3+6\mu^2d^3\lambda^2+3\mu^4d^3+4\lambda^2b_1\beta^2\right),$$
$$a_0=\frac{3}{8\lambda^2\beta^2}\left(\lambda^4d^2+2\lambda^2\mu^2d^2-4\lambda^2\mu d\beta+4\lambda^2\beta^2+\mu^4d^2-4\mu^3d\beta+4\beta^2\mu^2\right),\ a_1=0,\ a_2=0,\ b_1=b_1,$$
$$b_2=b_2; \tag{3.1.4}$$

Set 5

$$\lambda=\mu,\ \mu=\mu,\ \delta=\delta,\ \beta=0,\ W=0,\ d=d,\ a_{-2}=\frac{1}{\mu^2}\left(6d^4\Psi^2-\mu^2b_2+6\mu^2d^2+12\mu d^3\Psi\right),\ a_{-1}=\frac{1}{\mu^2}$$
$$\left(-12d^3\Psi^2-\mu^2b_1-\mu^26d-18\mu d^2\Psi\right),\ a_0=\frac{1}{\mu^2}\left(6d^2\Psi^2+\mu^2-\mu^26d-6\mu\delta d\right),\ a_1=0,\ a_2=0,\ b_1=b_1,$$
$$b_2=b_2; \tag{3.1.5}$$

where, $\Psi = \lambda - \delta$

Set 6

$$\lambda = -\mu,\ \mu = \mu,\ \delta = \delta,\ \beta = 0,\ W = 0,\ d = d,\ a_{-2} = \frac{1}{\mu^2}\left(6d^4\Psi^2 - \mu^2 b_2 + 6\mu^2 d^2 + 12\mu^2 d^3 - 12\delta\mu d^3\right),$$
$$a_{-1} = -\frac{1}{\mu^2}\left(12d^3\Psi^2 + \mu^2 b_1 + 6\mu^2 d - 18\mu d^2\Psi\right),\ a_0 = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d\right),\ a_1 = 0,\ a_2 = 0,$$
$$b_1 = b_1,\ b_2 = b_2; \qquad (3.1.6)$$

where, $\Psi = \lambda - \delta$

Set 7

$$\lambda = \lambda,\ \mu = \mu,\ \delta = \frac{1}{4\beta}\left(\lambda^2 + \mu^2 + 4\lambda\beta\right),\ \beta = \beta,\ W = 0,\ d = d,\ a_{-2} = -b_2,\ a_{-1} = -b_1,\ a_0 = \frac{3}{8\lambda^2\beta^2}\left(d^2\lambda^2\right.$$
$$\left.+2\lambda^2\mu^2 d^2 - 4\lambda^2\mu d\beta + 4\lambda^2\beta^2 + \mu^4 d^2 - 4\mu^3 d\beta + 4\beta^2\mu^2\right),\ a_1 = -\frac{3}{4\lambda^2\beta^2}\left(-2\mu\beta\lambda^2 - 2\beta\mu^3 + d\lambda^4 + 2d\lambda^2\mu^2\right.$$
$$\left.+\mu^4 d\right),\ a_2 = \frac{3}{8\lambda^2\beta^2}\left(\lambda^2 + \mu^2\right)^2,\ b_1 = b_1,\ b_2 = b_2; \qquad (3.1.7)$$

Set 8

$$\lambda = \lambda,\ \mu = \mu,\ \delta = -\frac{1}{4\beta}\left(\lambda^2 - \mu^2 - 4\lambda\beta\right),\ \beta = \beta,\ W = 0,\ d = d,\ a_{-2} = -b_2,\ a_{-1} = -b_1,\ a_0 = \frac{1}{8\lambda^2\beta^2}\left(3d^2\lambda^2\right.$$
$$\left.-6\lambda^2\mu^2 d^2 + 12\lambda^2\mu d\beta - 4\lambda^2\beta^2 + 3\mu^4 d^2 - 12\mu^3 d\beta + 12\beta^2\mu^2\right),\ a_1 = -\frac{3}{4\lambda^2\beta^2}\left(2\mu\beta\lambda^2 - 2\beta\mu^3 + d\lambda^4 -\right.$$
$$\left.2d\lambda^2\mu^2 + \mu^4 d\right),\ a_2 = \frac{3}{8\lambda^2\beta^2}\left(\lambda^2 - \mu^2\right)^2,\ b_1 = b_1,\ b_2 = b_2; \qquad (3.1.8)$$

Set 9

$$\lambda = \mu,\ \mu = \mu,\ \delta = \delta,\ \beta = 0,\ W = 0,\ d = d,\ a_{-2} = -b_2,\ a_{-1} = -b_1,\ a_0 = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 + 6\mu^2 d - 6\mu\delta d\right),$$
$$a_1 = \frac{1}{\mu^2}\left(-12d\Psi^2 - 6\mu\Psi\right),\ a_2 = \frac{6\Psi^2}{\mu^2},\ b_1 = b_1,\ b_2 = b_2; \qquad (3.1.9)$$

where, $\Psi = \lambda - \delta$

Set 10

$$\lambda = -\mu,\ \mu = \mu,\ \delta = \delta,\ \beta = 0,\ W = 0,\ d = d,\ a_{-2} = -b_2,\ a_{-1} = -b_1,\ a_0 = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d\right),$$
$$a_1 = \frac{1}{\mu^2}\left(-12d\Psi^2 + 6\mu^2 - 6\mu\delta\right),\ a_2 = \frac{6\Psi^2}{\mu^2},\ b_1 = b_1,\ b_2 = b_2; \qquad (3.1.10)$$

where, $\Psi = \lambda - \delta$

Set 11

$$\lambda=\lambda,\ \mu=-2d\Psi,\ \delta=\delta,\ \beta=-\frac{1}{16\Psi}\left(\lambda^2+16d^2\Psi^2\right),\ W=0,\ d=d,\ a_{-2}=-\frac{1}{128\Psi^2}\left(3\lambda^2+128b_2\Psi^2\right),$$

$$a_{-1}=-b_1,\ a_0=\frac{3}{4},\ a_1=0,\ a_2=\frac{6\Psi^2}{\lambda^2},\ b_1=b_1,\ b_2=b_2; \qquad (3.1.11)$$

where, $\Psi=\lambda-\delta$

Set 12

$$\lambda=\lambda,\ \mu=-2d\Psi,\ \delta=\delta,\ \beta=-\frac{1}{16\Psi}\left(-\lambda^2+16d^2\Psi^2\right),\ W=0,\ d=d,\ a_{-2}=-\frac{1}{128\Psi^2}\left(-3\lambda^2+128b_2\Psi^2\right),$$

$$a_{-1}=-b_1,\ a_0=\frac{1}{4},\ a_1=0,\ a_2=\frac{6\Psi^2}{\lambda^2},\ b_1=b_1,\ b_2=b_2; \qquad (3.1.12)$$

where, $\Psi=\lambda-\delta$

Non Travelling Wave Solutions

Substituting Eq. (3.1.1) in Eq. (3.3), along with Eq. (2.7) and simplifying, yields the following non-travelling wave solution, (if $C_1\neq 0$ and $C_2 = 0$)

$$\therefore u1_1(x,t)=\frac{3}{2}+\frac{3\lambda^2+4b_2\Psi^2}{2\Omega\tanh^2\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)},$$

Substituting Eq. (3.1.1) in Eq. (3.3), along with Eq. (2.8) and simplifying, our obtained solution become, (if $C_1\neq 0$ and $C_2 = 0$)

$$u1_2(x,t)=\frac{3}{2}+\frac{3\lambda^2+4b_2\Psi^2}{2\Omega i^2\tan^2\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)},$$

Substituting Eq (3.1.1) in Eq (3.3), along with Eq (2.9) and simplifying, our obtained solution become, (if $C_1\neq 0$ and $C_2 = 0$)

$$u1_3(x,t)=\frac{3}{2}+\frac{\left(3\lambda^2+4b_2\Psi^2\right)\left(C_1+C_2\xi\right)^2}{8\Psi^2C_2^2},$$

Similarly, substituting Eq. (3.1.1) in Eq. (3.3), along with Eq. (2.10) and simplifying, our obtained solution become, (if $C_1\neq 0$ and $C_2 = 0$)

$$u1_4(x,t)=\frac{3}{2}+\frac{3\lambda^2+4b_2\Psi^2}{8\left\{d\Psi+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2},$$

Substituting Eq. (3.1.1) in Eq. (3.3), along with Eq. (2.11) and simplifying, our obtained solution become, (if $C_1\neq 0$ and $C_2 = 0$)

$$u1_5(x,t)=\frac{3}{2}+\frac{3\lambda^2+4b_2\Psi^2}{8\left\{d\Psi-\sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2},$$

where $\xi = x - Wt$

Similarly, substituting Eq. (3.1.3) in Eq. (3.3), along with Eq. (2.7) - (2.11) and simplifying, our non-travelling wave solutions become:

$$u3_1(x,t) = a_0 + \frac{2\Psi(a_{-1}+b_1)}{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)} + \frac{4\Psi^2(a_{-2}+b_2)}{\left\{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2},$$

$$u3_2(x,t) = a_0 + \frac{2\Psi(a_{-1}+b_1)}{2d\Psi + \mu - \sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)} + \frac{4\Psi^2(a_{-2}+b_2)}{\left\{2d\Psi + \mu - \sqrt{\Omega}i\tan\left(\frac{\sqrt{-\Omega}}{2\Psi}\xi\right)\right\}^2},$$

$$u3_3(x,t) = a_0 + \frac{2\Psi(a_{-1}+b_1)(C_1+C_2\xi)}{2d\Psi(C_1+C_2\xi)+\mu(C_1+C_2\xi)+2\Psi C_2} + \frac{4\Psi^2(a_{-2}+b_2)(C_1+C_2\xi)^2}{\{2d\Psi(C_1+C_2\xi)+\mu(C_1+C_2\xi)+2\Psi C_2\}^2},$$

$$u3_4(x,t) = a_0 + \frac{\Psi(a_{-1}+b_1)}{d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)} + \frac{\Psi^2(a_{-2}+b_2)}{\left\{d\Psi + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{\Psi}\xi\right)\right\}^2},$$

$$u3_5(x,t) = a_0 + \frac{\Psi(a_{-1}+b_1)}{d\Psi - \sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)} + \frac{\Psi^2(a_{-2}+b_2)}{\left\{d\Psi - \sqrt{\Delta}i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2},$$

where $a_0 = \frac{1}{8\lambda^2\beta^2}\left(3\lambda^4 d^2 - 6\lambda^2\mu^2 d^2 + 12\lambda^2\mu d\beta - 4\lambda^2\beta^2 + 3\mu^4 d^2 - 12\mu^3 d\beta + 12\beta^2\mu^2\right)$,

$a_{-1} = -\frac{1}{4\lambda^2\beta^2}\left(36\mu^2\beta^2 d - 12\lambda^2\beta^2 d + 18\lambda^2 d^2\beta\mu - 18\mu^3\beta d^2 - 24\mu\beta^3 + 3\lambda^4 d - 6\mu^2 d^3\lambda^2 + 3\mu^4 d^3 + 4\lambda^2 b_1\beta^2\right)$,

$a_{-2} = \frac{1}{8\lambda^2\beta^2}\left(-24\lambda^2\beta^2 d^2 + 72\mu^2 d^2\beta^2 - 96\mu d\beta^3 - 8\lambda^2 b_2\beta^2 + 48\beta^4 + 3\lambda^4 d^4 - 6\lambda^2\mu^2 d^4 + 3d^4\mu^4 + 24\lambda^2\mu\beta d^3 - 24\mu^3\beta d^3\right)$,

$b_1 = b_1$, $b_2 = b_2$ and $\xi = x - Wt$.

Similarly, substituting Eq. (3.1.8) in Eq. (3.3), along with Eq. (2.7), (2.9) and (2.11) and simplifying, our obtained solutions become:

$$u8_1(x,t) = a_0 + \frac{2\Psi a_1}{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)} + \frac{4\Psi^2 a_2}{\left\{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2},$$

$$u8_2(x,t) = a_0 + \frac{a_1\{2d\Psi(C_1+C_2\xi)+\mu(C_1+C_2\xi)+2\Psi C_2\}}{2\Psi(C_1+C_2\xi)} + \frac{a_2\{2d\Psi(C_1+C_2\xi)+\mu(C_1+C_2\xi)+2\Psi C_2\}^2}{4\Psi^2(C_1+C_2\xi)^2},$$

$$u8_3(x,t) = a_0 + \frac{a_1\left\{d\Psi - \sqrt{\Delta}\, i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}}{\Psi} + \frac{a_2\left\{d\Psi - \sqrt{\Delta}\, i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right\}^2}{\Psi^2},$$

where $a_{-2} = -b_2,\ a_{-1} = -b_1,\ a_0 = \frac{1}{8\lambda^2\beta^2}\left(3d^2\lambda^2 - 6\lambda^2\mu^2 d^2 + 12\lambda^2\mu d\beta - 4\lambda^2\beta^2 + 3\mu^4 d^2\right.$ $\left.-12\mu^3 d\beta + 12\beta^2\mu^2\right),\ a_1 = -\frac{3}{4\lambda^2\beta^2}\left(2\mu\beta\lambda^2 - 2\beta\mu^3 + d\lambda^4 - 2d\lambda^2\mu^2 + \mu^4 d\right), a_2 = \frac{3}{8\lambda^2\beta^2}\left(\lambda^2-\mu^2\right)^2,\ b_1 = b_1,\ b_2 = b_2$

and $\xi = x - Wt$

Similarly, substituting Eq. (3.1.10) in Eq. (3.3), along with Eq. (2.7), (2.9) and (2.11) and simplifying, our non-travelling wave solutions become:

$$u10_1(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d - \frac{\left(6d\Psi^2 - 3\mu^2 + 3\mu\delta\right)\left\{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}}{\Psi} + \frac{3\left\{2d\Psi + \mu + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2\Psi}\xi\right)\right\}^2}{2}\right),$$

$$u10_2(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d - \frac{\left(6d\Psi^2 - 3\mu^2 + 3\mu\delta\right)\{2d\Psi(C_1+C_2\xi) + \mu(C_1+C_2\xi)+2\Psi C_2\}}{\Psi(C_1+C_2\xi)} + \frac{3\{2d\Psi(C_1+C_2\xi)+\mu(C_1+C_2\xi)+2\Psi C_2\}^2}{2(C_1+C_2\xi)^2}\right),$$

$$u10_2(x,t) = \frac{1}{\mu^2}\left(6d^2\Psi^2 + \mu^2 - 6\mu^2 d - 6\mu\delta d - \frac{12d\Psi^2 - 6\mu^2 + 6\mu\delta}{\Psi}\left(d\Psi - \sqrt{\Delta}\, i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right) + 6\left(d\Psi - \sqrt{\Delta}\, i\tan\left(\frac{\sqrt{-\Delta}}{\Psi}\xi\right)\right)^2\right),$$

where $\xi = x - Wt$

Discussions

Until now different methods have been used to different equations for constructing non-travelling wave solutions such as Zhang *et. al.,*[28] investigated by using generalized F- expansion method, Wang *et. al.*,[29] used the extended multiple Riccati equations expansion method, then Xie *et. al.*,[30] the improved extended tanh function method by generalizing the Riccati equation and so on. To our awareness the Fisher Equation is investigated to seek neither solutions for non-travelling wave nor exact solutions by the basic (G'/G) method. But in this paper we have used the new generalized and improved expansion (G'/G) method to Fisher Equation and it is important to point out that our obtained solutions are new, concise and direct and can be used for many other NLEEs.

Conclusions

In this article, the new extended generalized and improved method has been applied in the Fisher Equation successfully. In this method the auxiliary equation involving many arbitrary parameters and then the NLODE produces many new solutions. The solutions obtained by using this method, show that the method is effective and give concise and straightforward solutions. Therefore, it can be said that this method is powerful for constructing various types of wave and non-wave solutions of NLEEs those arise in every application of mathematical field.

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References

1. Hirota, R. (1973). Exact envelope-soliton solutions of a nonlinear wave equation. *Journal of Mathematical Physics*, 14(7), 805-809.
2. Malfliet, W. (1992). Solitary wave solutions of nonlinear wave equations. *American Journal of Physics*, 60(7), 650-654.
3. Abdou, M. A. (2007). The extended tanh method and its applications for solving nonlinear physical models. *Applied Mathematics and Computation*, 190(1), 988-996.
4. He, J. H., & Wu, X. H. (2006). Exp-function method for nonlinear wave equations. *Chaos, Solitons & Fractals*, 30(3), 700-708.
5. He, Y., Li, S., & Long, Y. (2012). Exact solutions of the Klein-Gordon equation by modified Exp-function method. *In Int. Math.* Forum (Vol. 7, No. 4, pp. 175-182).
6. Zhou, Y., Wang, M., & Wang, Y. (2003). Periodic wave solutions to a coupled KdV equations with variable coefficients. *Physics Letters A*, 308(1), 31-36.
7. Ali, A. T. (2011). New generalized Jacobi elliptic function rational expansion method. *Journal of computational and applied mathematics*, 235(14), 4117-4127.
8. Shi, L. M., Zhang, L. F., Meng, H., Zhao, H. W., & Zhou, S. P. (2011). A method to construct Weierstrass elliptic function solution for nonlinear equations. *International Journal of Modern Physics* B, 25(14), 1931-1939
9. Wang, M. (1995). Solitary wave solutions for variant Boussinesq equations. Physics letters A, 199(3-4), 169-172.
10. Wazwaz, A. M. (2004). Distinct variants of the KdV equation with compact and noncompact structures. *Applied Mathematics and Computation*, 150(2), 365-377.
11. Mohyud-Din, S. T., & Noor, M. A. (2007). Homotopy perturbation method for solving fourth-order boundary value problems. Mathematical Problems in Engineering, Article ID 98602, 15 pages doi:10.1155/2007/98602.
12. Soliman, A. A., & Abdo, H. A. (2012). New exact Solutions of nonlinear variants of the RLW, the PHI-four and Boussinesq equations based on modified extended direct algebraic method. arXiv preprint arXiv:1207.5127.
13. Rogers, C., & Shadwick, W. F. (1982). Bäcklund transformations and their applications. Academic press
14. Ma, W. X., & Fuchssteiner, B. (1996). Explicit and exact solutions to a Kolmogorov-

Petrovskii-Piskunov equation. *International Journal of Non-Linear Mechanics*, 31(3), 329-338.

15. Louis, H., Iserom, I. B., Akakuru, O. U., Nzeata-Ibe, N. A., Ikeuba, A. I., Magu, T. O., & Collins, E. O. (2018). L-state Solutions of the Relativistic and Non-Relativistic Wave Equations for Modified Hylleraas-Hulthen Potential Using the Nikiforov-Uvarov Quantum Formalism. *Oriental Journal of physical Sciences*, 3(1), 03-09.
16. Voskoglou, M. G. (2018). Solving Linear Programming Problems with Grey Data. *Oriental Journal of physical Sciences*, 3(1), 17-23.
17. Wang, M., Li, X., & Zhang, J. (2008). The (G′/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. Physics Letters A, 372(4), 417-423.
18. Zayed, E. M. E., & Gepreel, K. A. (2009). The (G′/G)-expansion method for finding traveling wave solutions of nonlinear partial differential equations in mathematical physics. *Journal of Mathematical Physics*, 50(1), 013502.
19. Feng, J., Li, W., & Wan, Q. (2011). Using G′G-expansion method to seek the traveling wave solution of Kolmogorov–Petrovskii–Piskunov equation. *Applied Mathematics and Computation*, 217(12), 5860-5865.
20. Naher, H., & Abdullah, F. A. (2012). The Basic (G'/G)-expansion method for the fourth order Boussinesq equation. *Applied Mathematics*, 3(10), 1144.
21. Zhang, J., Jiang, F., & Zhao, X. (2010). An improved (G′/G)-expansion method for solving nonlinear evolution equations. *International Journal of Computer Mathematics*, 87(8), 1716-1725.
22. Hamed, Y. S., Sayed, M., Elagan, S. K., & El-Zahar, E. R. (2011). The improved (G'/G)-expansion method for solving (3+1)-dimensional potential-YTSF equation. *Journal of Modern Methods in Numerical Mathematics*, 2(1-2), 32-39.
23. Naher, H., & Abdullah, F. A. (2012). Some New Traveling Wave Solutions of the Nonlinear Reaction Diffusion Equation by Using the Improved (G′/G)-Expansion Method. *Mathematical Problems in Engineering*, Article ID 871724, 17 pages, doi:10.1155/2012/871724.
24. Naher, H., & Abdullah, F. A. (2013). The Improved (G'/G)-Expansion Method to the (3 Dimensional Kadomstev-Petviashvili Equation. *American Journal of Applied Mathematics and Statistics*, 1(4), 64-70.
25. Naher, H., & Abdullah, F. A.. (2014). The improved (G'/G)-expansion method to the (2+1)-dimensional breaking soliton equation. *Journal of Computational Analysis & Applications*, 16(2), 220-235.
26. Naher, H., & Abdullah, F. A. (2013). New approach of (G′/G)-expansion method and new approach of generalized (G′/G)-expansion method for nonlinear evolution equation. AIP Advances, 3(3), 032116.
27. Naher, H., & Abdullah, F. A. (2016). Further extension of the generalized and improved (G′/G)-expansion method for nonlinear evolution equation. *Journal of the Association of Arab Universities for Basic and Applied Science*s, 19(1), 52-58.
28. Zhang, S., Li, W., Zheng, F., Yu, J. H., Ji, M., Lau, Z. Y., & Ma, C. Z. (2008). A Generalized F-expansion Method and its Application to (2+ 1)-dimensional Breaking Solition Equations. *International Journal of Nonlinear Science*, 5(1), 25-32.
29. Wang, D. S., & Li, H. (2008). Symbolic computation and non-travelling wave solutions of (2+ 1)-dimensional nonlinear evolution equations. Chaos, Solitons & Fractals, 38(2), 383-390.
30. Xie, F., Zhang, Y., & Lü, Z. (2005). Symbolic computation in non-linear evolution equation: application to (3+ 1)-dimensional Kadomtsev–Petviashvili equation. Chaos, Solitons & Fractals, 24(1), 257-263.