ISSN: 2456-799X, Vol.04, No.(1) 2019, Pg. 01-03

Oriental Journal of Physical Sciences

www.orientjphysicalsciences.org

Uncertainty, Fuzzy Sets and Related Theories

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Article History
Published on: 18 July 2019

Situations of uncertainty, which is connected to the shortage of precise knowledge and/or of complete information, appear frequently in problems of science, technology and in the everyday life. One may distinguish between two types of uncertainty characterized by *randomness* and *imprecision* respectively. In the former type the events and their outcomes are well defined, but the appearance of an outcome at a certain time cannot be predicted in advance. In the latter type of uncertainty the outcomes cannot be expressed in a crisp form, but they are described with linguistic characterizations. Half a century ago *probability theory* used to be the unique tool in hands of the experts for dealing with the uncertainty. However probability, which is based on principles of Aristotle's bi-valued logic, is limited to deal only with situations of uncertainty characterized by randomness.

Multi-valued logics have been studied since the 1920s by the Polish philosopher Jan Lukasiewicz, by the Polish-Jew logician and mathematician Alfred Tarski and by others. The electrical engineer of Iranian origin Lofti Zadeh, professor of computer science at the University of Berkley, California, introduced in 1965 *fuzzy sets* on the set of the discourse U.[1] A fuzzy set A on U is defined in terms of its *membership function* mapping each element x of U to a real number of the closed interval,[0, 1] called the *membership degree* of x in A. A crisp subset A of U can be considered as a fuzzy set on U with membership function taking only the values 1 if x is in A and 0 if x is not in A. The concept of fuzzy set gave genesis to *fuzzy logic*, an infinite-valued logic in which the truth values of variables range between completely true (1) and completely false (0). Fuzzy logic, which is a natural extension and generalization of the bi-valued logic, opened the door to the construction of mathematical solutions of problems stated in a natural language, in cases where probability theory has not the capability to do so. As a result it has rapidly expanded, its applications covering nowadays almost all sectors of the human activities; e.g. see Chapter 6 of,[2] Chapters 5-8 of,[3] etc.

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Doi: http://dx.doi.org/10.13005/OJPS04.01.01

Probabilities and membership degrees, despite to the fact that they act on the same real interval,[0, 1] they differ to each other in many aspects. For example, the statement "The probability of John to be tall is 85%" has a completely different meaning from the statement "John's membership degree in the fuzzy set of the tall people is 0.85". In fact, according to the Aristotle's principle of the excluded middle, the former statement means that John, being an unknown to the observer person, is either tall or short, but his outlines suggest that the probability to be tall is 85%. On the contrary, the latter statement means that John is a rather tall person. Another important difference between the two concepts is that the sum of the probabilities of all the single events (singleton subsets) of U is always 1 (probability of the certain event), but this is not necessarily true for the membership degrees. Zadeh introduced also the *fuzzy numbers* as a special form of fuzzy sets on the set of the real numbers and the basic arithmetic operations on them. Fuzzy numbers play an important role in fuzzy mathematics analogous to the role of the ordinary numbers in the traditional mathematics.

Fuzzy sets, apart from the randomness, handle effectively the kind of uncertainty termed as *vagueness*, where the information is naturally graded; e.g. "this colour is nearly red". However, there exist several other kinds of uncertainty caused by imprecision, which include *ambiguity* leading to several interpretations of the outcomes, *inconsistence* where two or more information cannot be true at the same time, etc. Many efforts have been made through the years by the experts to improve and generalize the fuzzy set theory on the purpose of handling better the existing uncertainty. The concept of the *interval-valued fuzzy set* was introduced in 1975 independently by Zadeh, Sambuc, Jahn and Grattan Guiness. An interval-valued fuzzy set is defined by a mapping from the universe U to the set of closed intervals in.[0, 1] The idea behind it is that the membership degrees of the traditional fuzzy sets can hardly be precise.

In 1975 Zadeh, in an effort to treat better the uncertainty about the membership function, introduced the *type-2 fuzzy set* in which the membership grades are themselves fuzzy. When no uncertainty exists about the membership function, then a type-2 fuzzy set reduces to an ordinary fuzzy set, which has been otherwise termed as a type-1 fuzzy set. The theory of type-2 fuzzy sets was further improved in the late 1990s by Prof. Jerry Mendel and his students and as a result more and more applications on it appear nowadays.

Kassimir Atanassov, professor of mathematics at the Bulgarian Academy of Sciences, introduced in 1986, as a complement of Zadeh's membership degree m(x), the degree of *non-membership* n(x) for all x in U and defined the *intuitionistic fuzzy set* where $0 \leq m(x) + n(x) \leq 1$. When the equality holds on the right hand side, we have an ordinary fuzzy set. This theory has been proved suitable for handling the existing imprecision in human thinking. Similar to the intention of the intuitionistic fuzzy set was the *vague set* introduced in 1993 by Gau and Buerher.

The Romanian–American mathematician Florentin Smarandache, professor at the branch of Gallup of the New Mexico University, introduced in 1995 the degree of *indeterminancy/neutrality* i(x) for all x in U and defined the *neutrosophic* set in three components m(x), n (x), i(x), which take values in the interval.[0, 1] This structure generalizes fuzzy sets and intuitionistic fuzzy sets and offers an effective framework to deal with neutral information (e.g. white votes in a ballot), which is not considered by the previous ones.

Complex fuzzy sets were introduced by Ramon, Milo, Friedman and Kandel in 2002, whose membership function is defined on the complex plane. In a *hesitant fuzzy set*, introduced by Torra and Narukawa in 2009, the *hesitant degree h*(x) of an element x of U is not a single value like its membership degree, but a set of some values in.[0, 1] On the other hand a *Pythagorean fuzzy set*, introduced by Yager in 2013, considers the membership degree *m*(x) and non-membership degree *n*(x) satisfying the condition $m^2(x) + n^2(x) \leq 1$. Hesitant fuzzy sets and Pythagorean fuzzy sets have stronger ability to manage the uncertainty in real-world decision-making problems.

Several related to fuzzy sets theories have been also proposed as alternative tools for managing the uncertainty in science, technology and in the everyday life. In 1982 Julong Deng, professor at the Huazhong University of Science and Technology, Wuhan, China, introduced the theory of *grey system* for handling the approximate data frequently appearing in the study of large and complex systems, like the socio-economic, the biological ones, etc. An effective tool for developing this theory that has found important applications in many fields of the human activity is the use of *Grey Numbers (GNs)* that are indeterminate numbers defined in terms of the closed real intervals.

A *rough set*, first described by the Polish computer scientist Zdzislaw Pawlak in 1991 is a formal approximation of a crisp set in terms of a pair of sets which give the *lower* and the *upper* approximation of the original set. In the standard version of rough set theory the lower and upper-approximation sets are crisp sets, but in other variations the approximating sets may be fuzzy sets. The theory of rough sets has found useful applications to Informatics and to other scientific fields.

In 1999 Dmtri Molodstov, Professor of the Computing Centre of the Russian Academy of Sciences in Moscow, in order to overcome the existing difficulty in defining the proper membership function, proposed the *soft sets* as a new mathematical tool for dealing with the uncertainties. Let E be a set of parameters, then a pair (F, E) is called a soft set on the universe U, if, and only if, F is a mapping of E into the set of all subsets of U. As an example, let U be the set of the girls of a high school and let E be the set of the characterizations {pretty, ugly, tall, short, clever} assigned to each of them. It becomes evident that for all ε in E the corresponding sets $F(\varepsilon)$ are arbitrary depending on the observer's personal criteria, while some of them could be empty or having non empty intersections. Fuzzy sets are special cases of soft sets.

The catalogue of the extensions and the related to fuzzy sets theories does not end here; several others have been also introduced through the years. In certain cases the corresponding concepts have been combined to form new hybrid generalizations and theories. For example, if in the definition of the soft set the set of all subsets of U is replaced by the set of all fuzzy subsets of U, one gets the notion of the *fuzzy soft set*, etc. For more details and for the ways of measuring the uncertainty in fuzzy systems the reader may look at.[2, 3]

Conclusion

Based on what it has been discussed in the present article, one concludes that there is not any general model which is ideal for handling the existing in the real world uncertainty. Everything depends on the form of the corresponding problem and on the given data. However, the combination of all the above mentioned theories offers a strong framework to be used for managing the several types and variations of the uncertainty in all cases.

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