ISSN: 2456-799X, Vol.06, No.(1-2) 2021, Pg. 01-03

Oriental Journal of Physical Sciences

www.orientaljphysicalsciences.org

Constant Sign Solutions of Nonlinear Boundary Value Problems Modeled by Ordinary and Partial Differential Equations

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Article History

Published on : 26 February 2022

The interest in this line of research lies in the importance of guaranteeing the existence of solutions of constant sign, for many of the physical quantities that appear in problems modeled by ODEs or PDEs. Such quantities can only take non-negative values (pressure, power, temperature in Kelvin degrees,...). If we focus, for example, on problems that model the deflection of supported (or embedded) beams (bridges) not only at the ends of the beam, but also at various intermediate points, the constant sign of the solution is equivalent to observing deviations from the equilibrium position in only one direction. This property is basic to keep the structure stable. Otherwise, we will be faced with the so-called buckling process, which can result in instability and the possible collapse of the structure, as happened in the famous Tacoma Bridge.

We highlight that the simplest stationary models that study the deviation of a beam consist of fourth-order differential equations dependent on a parameter that represents the force with which the structure reacts to deviations from its equilibrium position [DrLeHe]. The values of this parameter that guarantee that the beams move in the direction induced by the external force coincide with the constant sign solutions of the problem. This way, by obtaining the optimal values of the parameters for which the solution has constant sign, we can deduce the maximum values of the forces that the structure can support without buckling. Reciprocally, if we know a priori the forces in action, we can calculate the maximum distance allowed between the supports of a bridge to avoid buckling, as well as the best way to assemble the beam at its ends.

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Doi: http://dx.doi.org/10.13005/OJPS06.01-02.01

The intervals of definition of the considered problems can be bounded or unbounded. In the latter case we will deal with problems defined in the whole real line or in a half line, being able to study, in this case, the so-called traveling waves, which allow the modeling of certain types of tsunamis. Moreover, boundary value problems on unbounded intervals ascend in voluminous models of applied mathematics, such as combustion theory, plasma physics, models of unsteady flow of a gas through semi-infinite porous media or in study electrical potential of isolated neutral atom. For more specifics, techniques,& applications we refer, for instance, to [MiCa] and the monograph [AgOr].

Carrying out these studies is especially delicate and follows from the abstract formulation of the problem under consideration. The solutions of delinquent are the fixed points of an equivalent integral problem, these are determined by an integral kernel (Green's function) related to linear part of problem, the expression of which involves both the differential operator and the boundary conditions considered in each case. The fundamental properties of this type of functions as well as classic and recent results on the subject are compiled on the monograph [Ca]. In this work it is proved that the constant sign of Green's function is equivalent to fact that any solution of the studied problem has constant sign for any external force that does not change its sign.

The first complication that arises with this type of problems are the difficulty of the explicit calculus of Green's function. That is why an algorithm was developed in [CaCiMa], together with specific software, to calculate the expression of the Green's function of any linear BVP on bounded intervals http://library.wolfram.com/infocenter/MathSource/8825/.

The second problem that arises is that despite being able to have the unequivocal manifestation of Green's function, it is usually extremely complicated and, although it is true that its dependence on the parameters involved can be represented graphically and numerically studied, it is very difficult to ascertain analytically whether it has sign. Thus, to avoid the direct study of the expression of the Green's function in cases where it presents a particularly complicated expression, we mustfocus on the study on the qualitative properties of such functions. This study is related to the theory of disconjugacy [Co] which consists in guaranteeing that any solution of a linear ordinary differential equation, defined in a given interval, has no more zeros than the order of the equation. Thus, by combining this theory with the spectral one it has been characterized, for a huge class of boundary conditions, the set of parameters for which Green's function has persistent sign. Results are collected in [CaSa1, CaSa2], where the endpoints of this interval are characterized as the eigenvalues of certain related problems, whose expression is very easy to obtain and for which specific software was developed.

There are many open problems in this direction, both for local and nonlocal boundary conditions on the interval of definition.

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