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On The Efficiency of Ratio Estimators of Finite Population Mean using Auxiliary Information

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Abstract

Ratio estimation is technique that usages available auxiliary information which is certainly correlated with study variable. In this study, class of ratio-type estimators of finite population mean has been anticipated to solve delinquent of estimation of population mean. Properties of anticipated estimators namely Bias & Mean Square Error were acquired up to first order of approximation & condition for their efficiency over some existing estimators was also established. The results show that anticipated estimators are enhanced & proficient (minimum mean square errors) than other estimators with the highest precision.

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Introduction

Usage of auxiliary information is made through the ratio & product techniques of estimation to enhance estimates of population mean. Estimation of population mean of variable of interest with higher precision is unremitting issue in sample survey. So, precision could be increased by the used of apposite estimation procedure which consumes auxiliary information which is meticulously associated to variable of interest. In ratio method of estimation, auxiliary information is available which is linearly related to the variable of study. The population parameters such as populations' median, coefficient of kurtosis, skewness, coefficient of variation, decile, quartile, correlation, etc are auxiliary variables. Efficiency of estimators of population parameters can be increased by suitable usage of auxiliary information in relationship with auxiliary variable. Cochran[1] came up with what is known as ratio-type estimator for estimation of population mean which is more competent than sample mean. Many authors have used different auxiliary information inorder to

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enhance the precision of the estimates by using prior knowledge of population parameters. Researchers in sample survey like Kadilar and Cingi[2,3] developed classes of ratio estimators using known auxiliary information on coefficients of variation, & kurtosis. Abid *et al.*[4] also suggested set of ratio-type estimators for the population mean using non-conventional location parameters like mid-range, and tri-mean as auxiliary information. Other researchers are Upadhyaya and Singh,[5] Yan and Tian,[6] Subramani and Kumarapadiyan,[7] Subramani and Kumarapadiyan,[8] Subramani and Kumarapadiyan,[9] Subramani and Kumarapadiyan,[10] Jeelani *et al*,[11] and Nasir *et al.*[12]

The objective of this study is to develop innovative set of ratio-type estimators to increase precision of estimates of population mean using known auxiliary information.

Let $U=(U_1,U_2,\dots U_N)$ be finite population having N units & each $U_i=(X_i,Y_i)$, i = 1,2,3,...,N has pair of values. Y is study variable & X is auxiliary variable which is associated (correlated) with Y, in which $x = (x_1,x_2,\dots x_n)$ & $y = (y_1, y_2,\dots y_n)$ are the n sample values. & $\bar{y}$ is sample mean of variable of interest & $\bar{x}$ is sample mean of auxiliary variable. s_y^2 is sample mean square of study variable & s_x^2 is sample mean square of auxiliary variable based on random sample of size n drawn without replacement. and S_y^2 is population mean square of study variable and S_x^2 is population mean square of auxiliary variable. Following are other symbols used in this study.

Y : Study variable
N : Population size
X : Auxiliary variable
n : Sample size
$\bar{y}$: Sample means of study variable
$\bar{x}$: Sample means of auxiliary variable
$\bar{Y}$: Population means of study variable
$\bar{X}$: Population means of auxiliary variable
S_{pw} : Probability weighted moments
p : Coefficient of correlation
C_y : Coefficient of variation of study variable
Q_3 : The upper quartile
C_x : Coefficient of variation of auxiliary variable
QD : Population Quartile Deviation
β_1 : Coefficient of skewness
β_2 : Coefficient of kurtosis
G : Gini's Mean Difference
TM : Tri-Mean
M_d : Median
MR : Population mid-range
HR : Hodges-Lehman estimator
D : Downton's Method

$$\bar{X}=\frac{1}{N}\sum_{i=1}^{N}X_i,\quad \bar{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i,\quad \bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i,\quad \bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i,\quad \gamma=\frac{1-f}{n},$$

$$TM=\frac{(Q_1+2Q_2+Q_3)}{4},\ s_y^2=\frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2,\ s_x^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2,\ S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\bar{Y})^2,$$

$$S_x^2=\frac{1}{N-1}\sum_{i=1}^{N}(X_i-\bar{X})^2,\ MR=\frac{X_{(1)}+X_{(N)}}{2},\ HL=Median\left(\frac{(X_i+X_j)}{2},1\le i\le j\le N\right)$$

$$G=\frac{4}{N-1}\sum_{i=1}^{N}\left(\frac{2i-N-1}{2N}\right)X_i,\ D=\frac{2\sqrt{\pi}}{N(N-1)}\sum_{i=1}^{N}\left(i-\frac{N+1}{2}\right)X_i,\ S_{pw}=\frac{2\sqrt{\pi}}{N^2}\sum_{i=1}^{N}(2i-N-1)X_i$$

$$QD=\frac{Q_3-Q_1}{2}$$

The Existing Estimators in Literature

Cochran[1] developed the conventional ratio estimator for estimating population mean ($\bar{Y}$) of study variable (Y) given as:

$$\hat{\bar{Y}}_r=\frac{\bar{y}}{\bar{x}}\bar{X} \qquad ...(1.1)$$

where $R=\frac{\bar{Y}}{\bar{X}}$

$$Bias\left(\hat{\bar{Y}}\right)=\gamma\frac{1}{\bar{X}}\left(RS_x^2-\rho S_xS_y\right) \qquad ...(1.2)$$

$$MSE\left(\hat{\bar{Y}}_r\right)=\gamma\left(S_y^2+R^2S_x^2-2R\rho S_xS_y\right) \qquad ...(1.3)$$

Nasir *et al*[12] modified class of ratio type estimators for finite population mean consuming known values of coefficient of variation (C_x) & decile mean of auxiliary information, biases, constants and mean square errors are given as:

$$\hat{\bar{Y}}_1=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+DM)}(\bar{X}+DM) \qquad ...(1.4)$$

$$\hat{\bar{Y}}_2=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+DM)}(\bar{X}C_x+DM) \qquad ...(1.5)$$

$$\hat{\bar{Y}}_3=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+DM)}(\bar{X}\rho+DM) \qquad ...(1.6)$$

$$Bias\left(\hat{\bar{Y}}_i\right)=\gamma\frac{S_x^2}{\bar{Y}}R_i^2,\qquad where\ i=1,2,3 \qquad ...(1.7)$$

$$MSE\left(\hat{\bar{Y}}_i\right)=\gamma\left(R_i^2S_x^2+S_y^2\left(1-\rho^2\right)\right) \text{ where } i = 1,2,3 \qquad ...(1.8)$$

$$R_1=\frac{\bar{Y}}{\bar{X}+DM},\ R_2=\frac{\bar{Y}C_x}{\bar{X}C_x+DM},\ R_3=\frac{\bar{Y}\rho}{\bar{X}\rho+DM}$$

Subzar[13] developed a class ratio type estimators using linear combination of different known population parameters given as:

$$\hat{\bar{Y}}_4 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\psi_1)}(\bar{X}+\psi_1) \qquad \text{...(1.9)}$$

$$\hat{\bar{Y}}_5 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\psi_2)}(\bar{X}+\psi_2) \qquad \text{...(1.11)}$$

$$\hat{\bar{Y}}_6 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\psi_3)}(\bar{X}+\psi_3) \qquad \text{...(1.12)}$$

$$\hat{\bar{Y}}_7 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\psi_4)}(\bar{X}+\psi_4) \qquad \text{...(1.13)}$$

$$\hat{\bar{Y}}_8 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\psi_5)}(\bar{X}+\psi_5) \qquad \text{...(1.14)}$$

$$\hat{\bar{Y}}_9 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\psi_6)}(\bar{X}+\psi_6) \qquad \text{...(1.15)}$$

$$\hat{\bar{Y}}_{10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\psi_1)}(\bar{X}\rho+\psi_1) \qquad \text{...(1.16)}$$

$$\hat{\bar{Y}}_{11} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\psi_2)}(\bar{X}\rho+\psi_2) \qquad \text{...(1.17)}$$

$$\hat{\bar{Y}}_{12} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\psi_3)}(\bar{X}\rho+\psi_3) \qquad \text{...(1.18)}$$

$$\hat{\bar{Y}}_{13} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\psi_4)}(\bar{X}\rho+\psi_4) \qquad \text{...(1.19)}$$

$$\hat{\bar{Y}}_{14} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\psi_5)}(\bar{X}\rho+\psi_5) \qquad \text{...(1.21)}$$

$$\hat{\bar{Y}}_{15} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\psi_6)}(\bar{X}\rho+\psi_6) \qquad \text{...(1.22)}$$

$$\hat{\bar{Y}}_{16} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\psi_1)}(\bar{X}C_x+\psi_1) \qquad \text{...(1.23)}$$

$$\hat{\bar{Y}}_{17} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\psi_2)}(\bar{X}C_x+\psi_2) \qquad \text{...(1.24)}$$

$$\hat{\bar{Y}}_{18} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\psi_3)}(\bar{X}C_x+\psi_3) \qquad \text{...(1.25)}$$

$$\hat{\bar{Y}}_{19} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\psi_4)}(\bar{X}C_x+\psi_4) \qquad \text{...(1.26)}$$

$$\hat{\bar{Y}}_{20} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\psi_5)}(\bar{X}C_x+\psi_5) \qquad \text{...(1.27)}$$

$$\hat{\bar{Y}}_{21} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\psi_6)}(\bar{X}C_x+\psi_6) \qquad \text{...(1.28)}$$

where

$$\psi_1=(M_d+G),\ \psi_2=(M_d+D),\ \psi_3=(M_d+S_{pw}),\ \psi_4=(QD+G),\ \psi_5=(QD+D),\ \psi_6=(QD+S_{pw})$$

$$Bias\left(\hat{\bar{Y}}_j\right)=\gamma\frac{S_x^2}{\bar{Y}}R_j^2, \quad \text{Where j= 4,5,...,21} \qquad \text{...(1.29)}$$

$$MSE\left(\hat{\bar{Y}}_j\right)=\gamma\left(R_j^2S_x^2+S_y^2\left(1-\rho^2\right)\right) \quad \text{Where j= 4,5,..,21} \qquad \text{...(1.31)}$$

$$R_4=\frac{\bar{Y}}{\bar{X}+\psi_1},\ R_5=\frac{\bar{Y}}{\bar{X}+\psi_2},\ R_6=\frac{\bar{Y}}{\bar{X}+\psi_3} R_7=\frac{\bar{Y}}{\bar{X}+\psi_4},\ R_8=\frac{\bar{Y}}{\bar{X}+\psi_5},\ R_9=\frac{\bar{Y}}{\bar{X}+\psi_6},$$

$$R_{10}=\frac{\bar{Y}\rho}{\bar{X}\rho+\psi_1},\ R_{11}=\frac{\bar{Y}\rho}{\bar{X}\rho+\psi_2},\ R_{12}=\frac{\bar{Y}\rho}{\bar{X}\rho+\psi_3} R_{13}=\frac{\bar{Y}\rho}{\bar{X}\rho+\psi_4},\ R_{14}=\frac{\bar{Y}\rho}{\bar{X}\rho+\psi_5},\ R_{15}=\frac{\bar{Y}\rho}{\bar{X}\rho+\psi_6}$$

$$R_{16}=\frac{\bar{Y}C_x}{\bar{X}C_x+\psi_1},\ R_{17}=\frac{\bar{Y}C_x}{\bar{X}C_x+\psi_2},\ R_{18}=\frac{\bar{Y}C_x}{\bar{X}C_x+\psi_3} R_{19}=\frac{\bar{Y}C_x}{\bar{X}C_x+\psi_4},\ R_{20}=\frac{\bar{Y}C_x}{\bar{X}C_x+\psi_5},\ R_{21}=\frac{\bar{Y}C_x}{\bar{X}C_x+\psi_6}$$

Proposed Estimator

Motivated by the work of Subzar *et al*[13], we proposed ratio-type estimators for estimating population mean using value of hodges-lehmann as:

$$\hat{\bar{Y}}_{p1} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\varpi_1)}(\bar{X}+\varpi_1) \qquad \text{...(2.1)}$$

$$\hat{\bar{Y}}_{p2} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\varpi_2)}(\bar{X}+\varpi_2) \qquad \text{...(2.2)}$$

$$\hat{\bar{Y}}_{p3} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\varpi_3)}(\bar{X}+\varpi_3) \qquad \text{...(2.3)}$$

$$\hat{\bar{Y}}_{p4} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\varpi_4)}(\bar{X}+\varpi_4) \qquad \text{...(2.4)}$$

$$\hat{\bar{Y}}_{p5} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\varpi_5)}(\bar{X}+\varpi_5) \qquad \text{...(2.5)}$$

$$\hat{\bar{Y}}_{p6} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\varpi_6)}(\bar{X}+\varpi_6) \qquad \text{...(2.6)}$$

$$\hat{\bar{Y}}_{p7} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\varpi_1)}(\bar{X}\rho+\varpi_1) \qquad \text{...(2.7)}$$

$$\hat{\bar{Y}}_{p8} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\varpi_2)}(\bar{X}\rho+\varpi_2) \qquad \text{...(2.8)}$$

$$\hat{\bar{Y}}_{p9} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\varpi_3)}(\bar{X}\rho+\varpi_3) \qquad \text{...(2.8)}$$

$$\hat{\bar{Y}}_{p10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\varpi_4)}(\bar{X}\rho+\varpi_4) \qquad \text{...(2.11)}$$

$$\hat{\bar{Y}}_{p11} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\varpi_5)}(\bar{X}\rho+\varpi_5) \qquad \text{...(2.12)}$$

$$\hat{\bar{Y}}_{p12} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\varpi_6)}(\bar{X}\rho+\varpi_6) \qquad \text{...(2.13)}$$

$$\hat{\bar{Y}}_{p13} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\varpi_1)}(\bar{X}C_x+\varpi_1) \qquad \text{...(2.14)}$$

$$\hat{\bar{Y}}_{p14} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\varpi_2)}(\bar{X}C_x+\varpi_2) \qquad \text{...(2.15)}$$

$$\hat{\bar{Y}}_{p15} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\varpi_3)}(\bar{X}C_x+\varpi_3) \qquad \text{...(2.16)}$$

$$\hat{\bar{Y}}_{p16} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\varpi_4)}(\bar{X}C_x+\varpi_4) \qquad \text{...(2.17)}$$

$$\hat{\bar{Y}}_{p17} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\varpi_5)}(\bar{X}C_x+\varpi_5) \qquad \text{...(2.18)}$$

$$\hat{\bar{Y}}_{p18} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\varpi_6)}(\bar{X}C_x+\varpi_6) \qquad \text{...(2.19)}$$

where

$$Bias\left(\hat{\bar{Y}}_j\right)=\gamma\frac{S_x^2}{\bar{Y}}R_j^2, \quad \text{Where j= 4,5,..,21} \qquad \text{...(2.21)}$$

$$MSE\left(\ddot{\bar{Y}}_j\right)=\gamma\left(R_j^2S_x^2+S_y^2\left(1-\rho^2\right)\right) \quad \text{Where j= 4,5,..,21} \qquad \text{...(2.22)}$$

where

$$\varpi_1=(M_d+G\times S_{pw}),\ \varpi_2=(M_d+D\times S_{pw}),\ \varpi_3=(M_d+S_{pw}^2),\ \varpi_4=(QD+G\times S_{pw}),$$

$\varpi_5 = \left(QD + D \times S_{pw}\right),\ \varpi_6 = \left(QD + S_{pw}^2\right)$

In order to derive bias & mean square error, $e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$ and $e_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}$ such that

$\bar{y} = \bar{Y}\left(1+e_0\right)$ *and* $\bar{x} = \bar{X}\left(1+e_1\right)$, from the definition of e_0 and e_1, we obtain

$$\left.\begin{array}{l} E\left(e_0\right) = E\left(e_1\right) = 0,\ E\left(e_0^2\right) = \gamma C_y^2 \\ E\left(e_1^2\right) = \gamma C_x^2,\ E\left(e_0 e_1\right) = \gamma C_{yx} = \gamma \rho C_y C_x \end{array}\right\} \quad ...(2.23)$$

$$Bias\left(\hat{\bar{Y}}_{Pi}\right) = \gamma \frac{S_x^2}{\bar{Y}} R_{Pi}^2,\ \text{i= (1,2,3,4,5,6,)} \quad ...(2.24)$$

$$MSE\left(\hat{\bar{Y}}_{Pi}\right) = \gamma\left(R_{Pi}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right),\quad \text{i= (1,2,3,4,5,)} \quad ...(2.25)$$

where

$$R_{p1} = \frac{\bar{Y}}{\bar{X}+\varpi_1},\ R_{p2} = \frac{\bar{Y}}{\bar{X}+\varpi_2},\ R_{p3} = \frac{\bar{Y}}{\bar{X}+\varpi_3} R_{p4} = \frac{\bar{Y}}{\bar{X}+\varpi_4},\ R_{p5} = \frac{\bar{Y}}{\bar{X}+\varpi_5},\ R_{p6} = \frac{\bar{Y}}{\bar{X}+\varpi_6},$$

$$R_{p7} = \frac{\bar{Y}\rho}{\bar{X}\rho+\varpi_1},\ R_{p8} = \frac{\bar{Y}\rho}{\bar{X}\rho+\varpi_2},\ R_{p9} = \frac{\bar{Y}\rho}{\bar{X}\rho+\varpi_3} R_{p10} = \frac{\bar{Y}\rho}{\bar{X}\rho+\varpi_4},\ R_{p11} = \frac{\bar{Y}\rho}{\bar{X}\rho+\varpi_5},\ R_{p12} = \frac{\bar{Y}\rho}{\bar{X}\rho+\varpi_6}$$

$$R_{p16} = \frac{\bar{Y}C_x}{\bar{X}C_x+\varpi_1}, R_{p17} = \frac{\bar{Y}C_x}{\bar{X}C_x+\varpi_2}, R_{p18} = \frac{\bar{Y}C_x}{\bar{X}C_x+\varpi_3}, R_{p19} = \frac{\bar{Y}C_x}{\bar{X}C_x+\varpi_4}, R_{p20} = \frac{\bar{Y}C_x}{\bar{X}C_x+\varpi_5}, R_{p21} = \frac{\bar{Y}C_x}{\bar{X}C_x+\varpi_6}$$

Percentage Relative Efficiency (PRE)is given as

$$PRE = \frac{MSE\left(\hat{\bar{Y}}_r\right)}{MSE\left(\hat{\bar{Y}}_i\right)}\ X\ 100 \quad ...(2.26)$$

where $\hat{\bar{Y}}_j$ are estimators in this study.

Efficiency Comparisons

Efficiencies of suggested estimators are compared with efficiencies of existing estimators in study

$\hat{\bar{Y}}_{pi}$ - of proposed estimators of finite population mean is more efficient than $\hat{\bar{Y}}_r$ if,

$MSE\left(\hat{\bar{Y}}_{pi}\right) < MSE\left(\hat{\bar{Y}}_r\right)$ i= 1,2,...,18

$$\left(R_{pi}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right) < \left(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y\right) \quad ...(2.27)$$

The $\hat{\bar{Y}}_{pi}$ of proposed estimators of population mean is more efficient than if,

$MSE\left(\hat{\bar{Y}}_{pi}\right) < MSE\left(\hat{\bar{Y}}_j\right)$ i= 1,2,...,18 j= 1,2,3

$$\left(R_{pi}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right) < \left(R_j^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right) \quad ...(2.28)$$

When conditions (2.27), and (2.28) are contented, conclusion will be made that anticipated estimators are better and relatively efficient than other estimators in the study.

Empirical Study

To evaluate performance of anticipated estimators, following real populations are used.

Table 1: Characteristics of Populations [Subzar *et al.*[13]]

Parameter	Population I	Population II	Population III
N	34	34	80
n	20	20	20
$\bar{Y}$	856.4117	856.4117	5182.63
$\bar{X}$	199.4412	208.8823	1126.463
P	0.4453	0.4491	0.941
S_y	733.1407	733.1407	1835.659
C_y	0.8561	0.8561	0.354193
S_x	150.2150	150.5059	845.610
C_x	0.7531	0.7205	0.7506772
β_2	1.0445	0.0978	-0.063386
β_1	1.1823	0.9782	1.050002
M_d	142.5	150	757.5
MR	320	284.5	1795.5
TM	165.562	162.25	931.562
HL	320	190	1040.5
QD	184	80,25	588.125
G	162.996	155.446	901.081
D	144.481	140.891	801.381
S_{pw}	206.944	199.961	791.364
DM	206.944	234.82	1150.7

Table 2: Constant and Bias of Some Selected Existing and Proposed Estimators

Estimator	Constant			Bias		
	Pop-I	Pop-II	Pop-III	Pop-I	Pop-II	Pop-III
$\hat{\bar{Y}}_{r}$	4.294	4.100	4.601	4.940	4.270	60.877
$\hat{\bar{Y}}_{1}$	2.107	1.9301	2.276	2.137	2.2087	26.800
$\hat{\bar{Y}}_{2}$	1.806	1.6013	1.949	1.483	1.3964	19.650
$\hat{\bar{Y}}_{3}$	1.289	1.1703	2.206	0.800	0.7459	25.188
$\hat{\bar{Y}}_{4}$	1.6960	1.6651	1.9813	1.5604	1.5098	20.312
$\hat{\bar{Y}}_{5}$	1.7606	1.7136	2.0598	1.6815	1.599	21.953
$\hat{\bar{Y}}_{6}$	1.5602	1.5324	2.0681	1.3205	1.2788	22.129
$\hat{\bar{Y}}_{7}$	1.8955	1.9263	1.8608	1.9490	2.0207	17.916
$\hat{\bar{Y}}_{8}$	1.9764	1.9915	1.9299	2.1191	1.9915	2.1598
$\hat{\bar{Y}}_{9}$	1.7274	1.751	1.9371	1.6187	1.6696	19.416
$\hat{\bar{Y}}_{10}$	0.9671	0.9633	1.7938	0.5074	0.5053	16.650
$\hat{\bar{Y}}_{11}$	1.0148	0.9997	1.8622	0.5586	0.5443	17.942
$\hat{\bar{Y}}_{12}$	0.8701	0.8666	1.8693	0.4107	0.409	18.079
$\hat{\bar{Y}}_{13}$	1.1177	1.1672	1.9130	0.6777	0.7419	18.936
$\hat{\bar{Y}}_{14}$	1.1818	1.2211	1.9909	0.7577	0.8121	20.508
$\hat{\bar{Y}}_{15}$	0.9902	1.0283	1.9991	0.5318	0.5758	20.677
$\hat{\bar{Y}}_{16}$	1.4153	1.3533	1.5540	1.0866	0.9973	12.495
$\hat{\bar{Y}}_{17}$	1.4752	1.3979	1.6184	1.1806	1.0642	13.552
$\hat{\bar{Y}}_{18}$	1.2908	1.2329	1.6252	0.9038	0.8278	13.666
$\hat{\bar{Y}}_{19}$	1.6021	1.5977	1.6667	1.3923	1.3901	14.373
$\hat{\bar{Y}}_{20}$	1.6793	1.6603	1.7410	1.5298	1.5011	15.684
$\hat{\bar{Y}}_{21}$	1.4444	1.4326	1.7489	1.1317	1.1176	15.825
$\hat{\bar{Y}}_{p1}$	0.0005	0.0335	0.0061	0.0313	0.0006	0.0002
$\hat{\bar{Y}}_{p2}$	0.0007	0.0369	0.0069	0.0353	0.0007	0.0002
$\hat{\bar{Y}}_{p3}$	0.0003	0.0261	0.0070	0.0247	0.0004	0.0003
$\hat{\bar{Y}}_{p4}$	0.0005	0.0334	0.0062	0.0313	0.0006	0.0002
$\hat{\bar{Y}}_{p5}$	0.0007	0.0368	0.0069	0.0352	0.0007	0.0002
$\hat{\bar{Y}}_{p6}$	0.0003	0.0261	0.0070	0.0247	0.0004	0.0003
$\hat{\bar{Y}}_{p7}$	0.0005	0.0336	0.0062	0.0315	0.0006	0.0002
$\hat{\bar{Y}}_{p8}$	0.0007	0.0371	0.0069	0.0355	0.0007	0.0002
$\hat{\bar{Y}}_{p9}$	0.0003	0.0262	0.0070	0.0248	0.0004	0.0003
$\hat{\bar{Y}}_{p10}$	0.0005	0.0336	0.0062	0.0314	0.0006	0.0002
$\hat{\bar{Y}}_{p11}$	0.0007	0.0370	0.0069	0.0354	0.0007	0.0002
$\hat{\bar{Y}}_{p12}$	0.0003	0.0262	0.0070	0.0248	0.0004	0.0003
$\hat{\bar{Y}}_{p13}$	0.0005	0.0336	0.0062	0.0314	0.0006	0.0002
$\hat{\bar{Y}}_{p14}$	0.0007	0.0370	0.0069	0.0354	0.0007	0.0002
$\hat{\bar{Y}}_{p15}$	0.0003	0.0262	0.00701	0.0248	0.0004	0.0003
$\hat{\bar{Y}}_{p16}$	0.0005	0.0335	0.0062	0.0313	0.0006	0.0002
$\hat{\bar{Y}}_{p17}$	0.0007	0.0369	0.0069	0.0353	0.0007	0.0002
$\hat{\bar{Y}}_{p18}$	0.0003	0.0261	0.0070	0.0248	0.0004	0.0003

Table 2 shows the constant and bias of estimators

Table 3: MSE and PRE of Estimators

Estimator	MSE			PRE		
	Pop-I	**Pop-II**	**Pop-III**	**Pop-I**	**Pop-II**	**Pop-III**
$\hat{\bar{Y}}_r$	10960.76	10539.27	189775.1	100	100	100
$\hat{\bar{Y}}_1$	10934.74	10571.58	153292.6	100.238	99.69437	123.7993
$\hat{\bar{Y}}_2$	10386.83	10030.11	116239.3	105.5256	105.0763	163.2624
$\hat{\bar{Y}}_3$	9644.04	9472.95	144936.7	113.6532	111.2565	130.9365
$\hat{\bar{Y}}_4$	10208.16	10333.07	119741.5	107.3725	101.9955	158.4873
$\hat{\bar{Y}}_5$	10311.83	10203.59	128249.9	106.2931	103.2898	147.9729
$\hat{\bar{Y}}_6$	10002.72	9929.39	129161.3	109.5778	106.1422	146.9288
$\hat{\bar{Y}}_7$	10540.91	10564.74	107326.5	103.9831	99.75892	176.8204
$\hat{\bar{Y}}_8$	10686.6	10683.87	114349.5	102.5655	98.64656	165.9606
$\hat{\bar{Y}}_9$	10258.09	10264.06	115098.9	106.8499	102.6813	164.88
$\hat{\bar{Y}}_{10}$	9306.32	9266.94	100762.1	117.7776	113.7298	188.3398
$\hat{\bar{Y}}_{11}$	9350.19	9300.31	107457.3	117.225	113.3217	176.6051
$\hat{\bar{Y}}_{12}$	9223.53	9184.47	108172.8	118.8348	114.751	175.437
$\hat{\bar{Y}}_{13}$	9452.18	9469.56	112609.8	115.9601	111.2963	168.5245
$\hat{\bar{Y}}_{14}$	9250.7	9529.64	120761.3	118.4857	110.5946	157.1489
$\hat{\bar{Y}}_{15}$	9327.27	9327.31	121636.1	117.5131	112.9937	156.0187
$\hat{\bar{Y}}_{16}$	9802.37	9688.30	79228.58	111.8174	108.7835	239.5286
$\hat{\bar{Y}}_{17}$	9882.86	9745.56	84709.31	110.9068	108.1443	224.031
$\hat{\bar{Y}}_{18}$	9645.86	9543.10	85298.12	113.6318	110.4386	222.4845
$\hat{\bar{Y}}_{19}$	10064.19	10024.69	88963.27	108.9085	105.1331	213.3185
$\hat{\bar{Y}}_{20}$	10181.93	10119.77	95756.01	107.6491	104.1454	198.1861
$\hat{\bar{Y}}_{21}$	9841.01	9791.32	96489.29	111.3784	107.6389	196.68
$\hat{\bar{Y}}_{p1}$	8872.22	8834.676	14471.83	123.5402	119.2944	1311.341
$\hat{\bar{Y}}_{p2}$	8872.342	8834.787	14472.1	123.5385	119.2929	1311.317
$\hat{\bar{Y}}_{p3}$	8872.048	8834.471	14472.13	123.5426	119.2971	1311.314
$\hat{\bar{Y}}_{p4}$	8872.218	8834.674	14471.83	123.5402	119.2944	1311.341
$\hat{\bar{Y}}_{p5}$	8872.34	8834.786	14472.1	123.5385	119.2929	1311.317
$\bar{Y}_{p6}$	8872.047	8834.47	14472.13	123.5426	119.2971	1311.314
$\hat{\bar{Y}}_{p7}$	8872.223	8834.68	14471.83	123.5402	119.2943	1311.341
$\hat{\bar{Y}}_{p8}$	8872.348	8834.794	14472.1	123.5384	119.2928	1311.317
$\hat{\bar{Y}}_{p9}$	8872.05	8834.473	14472.13	123.5426	119.2971	1311.314
$\hat{\bar{Y}}_{p10}$	8872.222	8834.679	14471.83	123.5402	119.2943	1311.341
$\hat{\bar{Y}}_{p11}$	8872.346	8834.792	14472.1	123.5385	119.2928	1311.317
$\hat{\bar{Y}}_{p12}$	8872.049	8834.472	14472.13	123.5426	119.2971	1311.314
$\hat{\bar{Y}}_{p13}$	872.221	8834.678	14471.83	123.5402	119.2943	1311.341
$\hat{\bar{Y}}_{p14}$	8872.345	8834.791	14472.1	123.5385	119.2928	1311.317
$\hat{\bar{Y}}_{p15}$	8872.049	8834.472	14472.13	123.5426	119.2971	1311.314
$\hat{\bar{Y}}_{p16}$	8872.22	8834.677	14471.83	123.5402	119.2943	1311.341
$\hat{\bar{Y}}_{p17}$	8872.343	8834.789	14472.1	123.5385	119.2928	1311.317
$\hat{\bar{Y}}_{p18}$	8872.048	8834.471	14472.13	123.5426	119.2971	1311.314

Table 3 : shows the mean square error (MSE) & percentage relative efficiency (PRE) for the three populations.

Results & Discussion

Class of ratio estimators of finite population mean is proposed and performance of anticipated estimators over existing estimators were established. The scope of the study is to analyze and estimate the biasness, mean square errors of anticipated estimators and efficiency comparison with some existing estimators. Tables 2 and 3 show the results of the Constant, Bias, Mean Square Error (MSE) & Percentage Relative Efficiency (PRE) of anticipated & existing estimators considered in study for all populations used. Outcomes also discovered that anticipated estimators have least MSE and advanced PRE than other estimators. The outcomes also show that average dispersion of anticipated estimators gives better estimates on the average compare to other estimators considered.

Future Scope

The future scope ofstudy is to transform the sampling technique from simple random sampling to other sampling techniques like stratified sampling, two stage sampling, cluster sampling or successive sampling.

Conclusion

In Table 3, anticipated estimators performed better than prevailing estimators considered in study. So, it is clear that anticipated estimators performed superior than other estimators having minimum Mean Square Error (MSE) & highest Percentage Relative Error (PRE). We therefore conclude that anticipated estimators are relatively efficient and better than other estimators for estimation of population mean.

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Conflict of Interest

The authors declare no conflict of interest.

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