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On 'Useful' R-norm Relative Information Measures and Applications

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Abstract

In this communication a new 'useful' R-norm relative information measure is introduced and characterized axiomatically. Its in equalities with particular cases are described. This new information measure has also been applied to study the status of the companies with regard to their loss and profit and that has been illustrated by considering empirical data and drawing figures. Ad joint of the relative information measure is also defined with the illustration of its application in share market with examples.

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Introduction

Information theory as a separate subject is about 70 years old. Since information is energy, therefore it is measured, managed, regulated and controlled for the sake of welfare of humanity. The role of information function is to remove uncertainty and the amount of uncertainty removed is a measure of information.

The concept of information proved to be very important and universally useful. These days language used in telephones, business management, and cybernetics falls under the name "Information Processing". In addition to this, information theory particularly measures of information have applications in physics, statistical inference, data processing and analysis, accountancy, psychology, etc.

Shannon[24] was the first who developed a measure of uncertainty. He was interested in communicating information across the channel

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in which some information is lost in the process of communication and that was called a noisy channel. His objective was to measure the amount of information lost. He defined a measure of uncertainty of a probability distribution as given below:

$$H(P)= -k\Sigma P_i \log P_i, \quad ...(1.1)$$

where k is an arbitrary positive constant. The measure(1.1) was called entropy. Thereafter, Shannon's entropy wascharacterized by various researchers like Khinchin,[16] Fadeev.[8] Teverberg,[25] Chandy and Mcleod,[5] Kendal,[15] Lee,[20] Berges,[2] Cziszar,[7] Cheng,[6] etc. on using different sets of postulates.

The quantity (1.1) measures the amount of information of probability distribution P when effectiveness or importance of the events is not taken into account. In addition to this; some probabilistic problems also play important role. Considering effectiveness of the outcomes, Belis and Guiasu[1] introduced U= $(u_1,u_2,\ldots,u_n)$ as a utility distribution, where $u_i>0$, is the usefulness of an event having probability of occurrence pi and consequently, "self useful information' is defined as given below:

$$H(u_i, P_i) =-u_i^{\log} P_i. \quad ...(1.2)$$

The measure (1.2) is based two postulated as given below:

P1. In case all the events of an random experiment have the same utility u>0, then the self used information generated by the product of two statistical independent events E_1 and E_2 can be expressed as the sum of the self-useful information provided by E_1 and E_2 individually i.e.

$$H(p_1{*}\ p_2; u) = H(p_1; u)+ H(P_2;u), \quad ...(1.3)$$

where $p_1{*}\ p_2$ is the probability of $E_1 \cap E_2$

p2. $H(p_i; \delta u) = \delta H(p_i;u_i)$ for $\delta>0$. ...(1.4)

Further, Belis and Gauisu [1] gave the following qualitative and qualitative information measure:

$$H(U;P)=- , -\sum_{i=1}^{n} u_i p_i \log p_i ; where\ u_i>0\ and \sum_{i=1}^{n} p_i=1, \quad ...(1.5)$$

Longo[21] called (1.5) as 'useful' information and Guiasu and Picard[9] called weighted entropy.

In this communication , the 'useful' relative information measure is defined and characterized axiomatically in section 2. The new measure thus introduced is generalized in section 3 with its and its particular cases are studied in section 4. The applications of new R-norm information measure are described in section 5. In section 6 its ad joint by taking empirical data is studied with its illustration graphically. In the end the conclusion is given along with an exhaustive list of references.

Useful' Relative Information Measure

Let X be a random variable in an experiment and

$$\left\{P=(p_1,p_2,\ldots,p_n), where p_i \geq 0 \text{ and} \sum_{i=1}^{n} p_i=1\right\}$$

be its probability distribution having U=(u_1, u_2........, u_n) as a utility distribution, where ui>0 for each i, is the utility of an event having probability p_i .

A 'useful' directed divergence measure was defined by Bhaker and Hooda [3] and characterized as given below:

$$D(U;P:Q)=\frac{\sum_{i=1}^{n} u_i p_i \log\left(\frac{p_i}{q_i}\right)}{\sum_{i=1}^{n} u_i p_i}, \quad \text{where}$$

$p_i \geq 0$ and $q_i>0$ s.t. $\sum_{i=1}^{n} pi = \sum_{i=1}^{n} qi$ ...(2.1)

If we consider a uniform probability $Q=\left\{\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\right\}$ in (2.1), it reduces to log *n*- (U;P),

where

$$H(P;U)=-\frac{\sum_{i=1}^{n} u_i p_i \log p_i}{\sum_{i=1}^{n} u_i p_i}. \quad ...(2.2)$$

It may be noted that 'useful' directed divergence measure D(U;P:Q) satisfies the following conditions:

1. D(U;P:Q) $\geq$ 0
2. D(U;P:Q) = 0 iff for each i, $P_i = q_i$.
3. D(U;P:Q) is a convex function of q_1, q_2,.... P_n. as well as p_1, p_2,.... p_n.

Further, it is observed that (1.1) is not symmetric in P and Q, since D(U;P:Q) $\neq D(U;Q:P)$.
Later on a symmetric 'useful' J- divergence measure was introduced by Hooda and Ram [2] as given below:

$$J(U;P:Q)=D(U;P:Q)+D(U;Q:P)$$

$$=\frac{\sum_{i=1}^{n}u_ip_i\log\left(\frac{p_i}{q_i}\right)}{\sum_{i=1}^{n}u_ip_i}+\frac{\sum_{i=1}^{n}u_iq_i\log\left(\frac{q_i}{p_i}\right)}{\sum_{i=1}^{n}u_iq_i}$$

$$=\frac{\sum_{i=1}^{n}u_i(p_i-q_i)\log\left(\frac{p_i}{q_i}\right)}{\sum_{i=1}^{n}u_ip_i}, \quad ...(2.3)$$

where $\sum_{i=1}^{n}u_ip_i=\sum_{i=1}^{n}u_iq_i$. In case u_i = 1 for $\forall\, i$, then (2.3) reduces to

$$J(P:Q)=\sum_{i=1}^{n}(p_i-q_i)\log\left(\frac{p_i}{q_i}\right), \quad ...(2.4)$$

where (2.4) is a divergence measure is due to Jeffrey and thus it is called as J-divergence.

Bhaker and Hooda [3] also characterized a 'useful' relative information measure of order α as given below.

$$D_\alpha(P:Q;U)=(\alpha-1)^{-1}\log\frac{\sum_{i=1}^{n}u_ip_i^{\alpha}q_i^{\alpha-1}}{\sum_{i=1}^{n}u_ip_i}. \quad ...(2.5)$$

Further Hooda and Ram [10] characterizednon-additive 'useful' relative information of degree β as given below:

$$D^\beta(P:Q;U)=\frac{1}{2^{\beta-1}-1}\left[\frac{\sum_{i=1}^{n}u_ip_i^{\beta}q_i^{1-\beta}}{\sum_{i=1}^{n}u_ip_i}-1\right]. \quad ...(2.6)$$

The measure (2.6) reduces to (2.1) when , while in case u_i=1then(2.6)reduces to Kullback-Leibler's [17]'useful' relative information measure.

Further Boekee and Lubbe [4] defined R-norm information of the distribution P as

$$H_R(P)=\frac{R}{R-1}[1-(\sum_{i=1}^{n}p_i^{R})^{\frac{1}{R}}] \quad ...(2.7)$$

Kumaret al.[18] also defined the following 'useful' R-norm relative measure:

$$D_R(U;P:Q)=\frac{R}{1-R}\left[1-\left(\frac{\sum_{i=1}^{n}u_ip_i^{R}q_i^{1-R}}{\sum_{i=1}^{n}u_ip_i}\right)^{\frac{1}{R}}\right], where R>0(\neq 1). \quad ...(2.8)$$

There are many other generalizations of (2.8) also and one of them is

$$D_R^\beta(P:Q;U)=\frac{R}{\beta-R}\left[1-\left(\frac{\sum_{i=1}^{n}u_ip_i^{R-\beta+1}q_i^{\beta-R}}{\sum_{i=1}^{n}u_ip_i}\right)^{\frac{1}{R-\beta+1}}\right], R>0(\neq 1),\ 0<\beta\leq 1, \quad .(2.9)$$

where $\sum_{i=1}^{n}u_ip_i\geq\sum_{i=1}^{n}u_iq_i$.

A 'Useful' R-Normrelative Information Measure.
Theorem 3.1

Let P and Q be two probability distributions attached with a utility distribution U, then the following holds:

$$\left(\frac{\sum_{i=1}^{n}u_ip_i^{R-\beta+1}q_i^{\beta-R}}{\sum_{i=1}^{n}u_ip_i}\right)^{\frac{1}{R-\beta+1}} \gtreqless 1 \text{ according to } R\gtreqless\beta$$

under the condition $\sum_{i=1}^{n}p_iu_i\geq\sum_{i=1}^{n}q_iu_i$

Proof: We know from Holder's inequality as

$$\left(\sum_{i=1}^{n}x_i^{p}\right)^{\frac{1}{p}}\left(\sum_{i=1}^{n}y_i^{q}\right)^{\frac{1}{q}}\leq\sum_{i=1}^{n}x_iy_i \text{, where } R>\beta \quad ...(3.1)$$

$x_i=u_i^{R-\beta+1}p_i^{R-\beta+1}, y_i=u_i^{\beta-R}q_i^{\beta-R}$, Setting

and $\frac{1}{p}+\frac{1}{q}=1\,; p=\frac{1}{R-\beta+1}, q=\frac{1}{\beta-R}, x_i,y_i\geq 0$.

In Holder's inequality (3.1), we have

$$\left[\sum_{i=1}^{n}\left(u_i^{R-\beta+1}p_i^{R-\beta+1}\right)^{\frac{1}{R-\beta+1}}\right]^{R-\beta+1}\left[\sum_{i=1}^{n}\left(u_i^{\beta-R}q_i^{\beta-R}\right)^{\frac{1}{\beta-R}}\right]^{\beta-R}\leq\sum_{i=1}^{n}u_ip_i^{R-\beta+1}q_i^{\beta-R}$$

On simplification we have

$$\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R} \geq \left(\sum_{i=1}^{n} u_i p_i\right)^{R-\beta+1} \left(\sum_{i=1}^{n} u_i q_i\right)^{\beta-R} \geq \sum_{i=1}^{n} u_i p_i \,,$$

since $\sum_{i=1}^{n} p_i u_i \geq \sum_{i=1}^{n} q_i u_i$

or

$$\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i} \geq 1 \,. \qquad ...(3.2)$$

Since $\frac{1}{R-\beta+1} < 1$, therefore (3.2) can be written as

$$\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}} \geq 1 \,, R > \beta \,. \qquad ...(3.3)$$

Similarly, we can prove that

$$\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}} \leq 1 \text{ , for } R< \beta$$

and $\frac{1}{R-\beta+1} > 1$. ...(3.4)

For $R = \beta$, $\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}} = 1$ for all probability distributions and if $R \neq \beta$ and $p_i = q_i$, for each i, i.e. P = Q, then we have

$$\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}} = 1 \,. \qquad ...(3.5)$$

Theorem 2.2

$D^{\beta}_{R}(U;P:Q)$ is a convex function of P and Q.

Proof

Let $K = \frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}$. On differentiating K with regard to p_i with all q^i and u_i having fixed value, we have $\sum_{i=1}^{n} u_i q_i$ fixed and consequently,

$\sum_{i=1}^{n} u_i p_i \geq \sum_{i=1}^{n} u_i q_i$ is constant.

Hence we can write $K = T\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}$,

where $T = \frac{1}{\sum_{i=1}^{n} u_i p_i}$ = Constant.

It implies that

$$\frac{\partial K}{\partial p_i} = T u_i (R-\beta+1) p_i^{R-\beta} q_i^{\beta-R} \qquad ...(3.6)$$

or

$$\frac{\partial^2 K}{\partial p_i^2} = T u_i (R-\beta+1)(R-\beta) p_i^{R-\beta-1} q_i^{\beta-R} \qquad ...(3.7)$$

or R-β >0, (2.6) is positive.

It implies that $\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}$ and $\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}}$ are convex functions of P in view of $\frac{1}{R-\beta+1} < 1$.

Similarly for, $\frac{R}{R-\beta} < 0$, $\frac{R}{\beta-R}\left[1-\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}}\right]$ is also aconvex function of P.

For R-β<0, (3.6) is negative, so $\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}$ and $\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}}$ are concave functions of P, since $\frac{1}{R-\beta+1}>1$.

Hence for R-β<0 & R-β<0, D^{β}_{R}(U;P: Q) is also a convex function of P.

On same lines we can prove that D^{β}_{R}(U;P: Q) is a convex function of Q for R-β<0 and R-β>0 provided

$\sum_{i=1}^{n} u_i p_i \geq \sum_{i=1}^{n} u_i q_i$.

A Generalized 'Useful' R-norm Relative information Measure of Degree β

We consider the following function:

$$D_R^{\beta}(P:Q;U)=\frac{R}{\beta-R}\left[F(1)-F\left\{\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}}\right\}\right], \quad ...(4.1)$$

where F (x) is a monotonic increasing function of x.

In view of (3.2), R-β>0 and $\frac{1}{R-\beta+1}<1$, we have

$$\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}} \geq 1. \quad ...(4.2)$$

It implies

$$F\left\{\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}}\right\} \geq F(1) \quad ...(4.3)$$

Or

$$F(1)-F\left\{\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}}\right\} \leq 0 \quad ...(4.4)$$

Multiplying (4.3) by $\frac{R}{\beta-R}<0$, we get

$$\frac{R}{\beta-R}\left[F(1)-F\left\{\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}}\right\}\right] \geq 0$$

or

$$D_R^{\beta}(P:Q;U) \geq 0. \quad ...(4.5)$$

Similarly, in view of R-β<0, and $\frac{1}{R-\beta+1}>1$, (4.4) can be written as

$$\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}} \leq 1 \quad ...(4.6)$$

It implies that

$$F\left\{\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}}\right\} \leq F(1) \quad ...(4.7)$$

or

$$F(1)-F\left\{\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}}\right\} \geq 0 \quad ...(4.8)$$

Multiplying (4.8) by $\frac{R}{\beta-R}<0$, we get

$$\frac{R}{\beta-R}\left[F(1)-F\left\{\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}}\right\}\right] \geq 0 \quad ...(4.9)$$

or

$$D_R^{\beta}(P:Q;U) \geq 0. \quad ...(4.10)$$

Hence from (4.5) and (4.6) together give

$$D_R^{\beta}(P:Q;U) \geq 0 \qquad D_R^{\beta}(P:Q;U) \geq 0 \quad ...(4.11)$$

It may be noted that (4.11) vanishes when $p_i = q_i$ for each *i*.

It implies that

$$\frac{R}{\beta-R}\left[F(1)-F\left\{\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}}\right\}\right] 0, \text{ when } p_i=q_i$$

for each i

or

$$F\left\{\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}}\right\} = F(1)$$

or

$$\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} q_i^{\beta-R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}} = 1$$ when $p_i=q_i$ for each i. ...(4.12)

In particular when β=1 and R→1, then $D_R^{\beta}(P:Q;U)$ reduces to $\frac{\sum_{i=1}^{n} u_i p_i \log \frac{p_i}{q_i}}{\sum_{i=1}^{n} u_i p_i}$ which is (2.1)

Particular Cases

When $C=\left(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\right)$, then (4.11) reduces to

$$D(P:C;U)=\frac{\sum_{i=1}^{n} u_i p_i \log np_i}{\sum_{i=1}^{n} u_i p_i} \geq 0, \qquad ...(4.13)$$

and in case P = C; D(P;C;U) =0,.
Further, it can be verified that D(P:C;U) is a convex function of P.
Next, it may be noted that (4.13) can be written as

$$D(P:C;U)=\log n+\frac{\sum_{i=1}^{n} u_i p_i \log p_i}{\sum_{i=1}^{n} u_i p_i}=D(C;U)-D(P;U).$$

...(4.14)

Next we consider

$$H_{R,F}^{\beta}(P;U)=\frac{R}{\beta-R}\left[F\left\{\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} n^{R-\beta}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R-\beta+1}}\right\}-F\left(n^{\frac{R-\beta}{R-\beta+1}}\right)\right],$$

or

$$H_{R,F}^{\beta}(C;U)=\frac{R}{\beta-R}\left[F(1)-F\left(n^{\frac{R-\beta}{R-\beta+1}}\right)\right] \qquad ...(4.15)$$

and

$$\lim_{R\leftarrow 1} H_{1,F}^{1}(C;U)=F'(1)\log n. \qquad ...(4.16)$$

Further, if $F(x)=x^j, (j\geq 1)$, then we have

$$H_{R,j}^{\beta}(P;U)=\frac{R}{\beta-R}\left[\left(\frac{\sum_{i=1}^{n} u_i p_i^{R-\beta+1} n^{R-\beta}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{j}{R-\beta+1}}-\left(n^{R-\beta}\right)^{\frac{j}{R-\beta+1}}\right]$$

$$=\left(n^{R-1}\right)^{\frac{j}{R}}\frac{R}{1-R}\left[\left(\frac{\sum_{i=1}^{n} u_i p_i^{R} n^{R-1}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{j}{R}}-1\right], \text{ when } \beta=1. \qquad ...(4.17)$$

For j = 1, we get

$$H_{R,1}^{1}=n^{\frac{R-1}{R}}\frac{R}{1-R}\left[\left(\frac{\sum_{i=1}^{n} u_i p_i^{R}}{\sum_{i=1}^{n} u_i p_i}\right)^{\frac{1}{R}}-1\right], \qquad ...(4.18)$$

In case R→1, the measure (4.13),(4.17) and (4.5) respectively reduce to

$$H_{1,F}^{1}(P;U)=-F'(1)\left(\frac{\sum_{i=1}^{n} u_i p_i \log p_i}{\sum_{i=1}^{n} u_i p_i}\right); \beta=1, \qquad ...(4.19)$$

$$H_{1,j}^{1}(P;U)=-j\frac{\sum_{i=1}^{n} u_i p_i \log p_i}{\sum_{i=1}^{n} u_i p_i}, \qquad ...(4.20)$$

and

$$H_{1,1}^{1}(P;U)=-\frac{\sum_{i=1}^{n} u_i p_i \log p_i}{\sum_{i=1}^{n} u_i p_i}, \qquad ...(4.21)$$

It may be noted that (4.21) was defined and characterized by Bhaker and Hooda [3].
In case F(x) = logx in (4.1), it reduces to

$$H^{\beta}_{R,F}(P;U)=\frac{R}{\beta-R}\left[\log\left\{\left(\frac{\sum_{i=1}^{n}u_i p_i^{R-\beta+1}n^{R-\beta}}{\sum_{i=1}^{n}u_i p_i}\right)^{\frac{1}{R-\beta+1}}\right\}-\log\left(n^{\frac{R-\beta}{R-\beta+1}}\right)\right] \qquad ...(4.22)$$

or

$$H^{1}_{R,F}=\frac{1}{1-R}\log\left(\frac{\sum_{i=1}^{n}u_i p_i^{R}}{\sum_{i=1}^{n}u_i p_i}\right), \text{ when } \beta=1. \qquad ...(4.23)$$

In case $u_i = 1$ for each i in (4.10), it reduces

$$H_{R,F}(P;U)=\frac{1}{1-R}\log\sum_{i=1}^{n}p_i^{R}, \qquad ...(4.24)$$

which is well known Renyi's [23] entropy of order R.

Illustration with an Example

In this section we consider production data of different companies due to Nager and Singh[22] represented in Table 5.1. We calculate D^{β}_{R} (P:Q;U) in Table 5.2 and represent graphically in fig.5.1.

Table 5.1: Data due to Nager and Singh[22]

S.No	Company's Name	2 Sept 2010	pi	3 Sept 2010	q_i	u_i
1	Reliance Ind.	10.46	0.1046	10.44	0.1044	30
2	Infosys Tech.	10.04	0.1004	9.93	0.0993	29
3	ICICI Bank	8.38	0.0838	8.43	0.0843	28
4	L&T	8.37	0.0837	8.39	0.0839	27
5	ITC	6.41	0.0641	6.47	0.0647	26
6	HDFC	6.05	0.0605	6.13	0.0613	25
7	HDFC Bank	6.02	0.0602	6.10	0.061	24
8	SBI	5.40	0.054	5.35	0.0535	23
9	ONGC	3.40	0.034	3.36	0.0336	22
10	Bharti Airtel	3.18	0.0318	3.14	0.0314	21
11	Tata Consult	3.01	0.0301	2.96	0.0296	20
12	BHEL	2.89	0.0289	2.85	0.0285	19
13	Tata Steel	2.42	0.0242	2.44	0.0244	18
14	Tata Motors	2.29	0.0229	2.30	0.023	17
15	Hindustan Unilever	2.14	0.0214	2.15	0.0215	16
16	Jindal Steel	2.03	0.0203	2.02	0.0202	15
17	M&M	1.94	0.0194	1.94	0.0194	14
18	Hindako	1.61	0.0161	1.61	0.0161	13
19	Sterlite Industry	1.57	0.0157	1.59	0.0159	12
20	Wipro	1.54	0.0154	1.54	0.0154	11
21	Tata Power	1.48	0.0148	1.50	0.015	10
22	NTPC	1.29	0.0129	1.28	0.0128	9
23	Maruti Suzuki	1.27	0.0127	1.27	0.0127	8
24	Hero Honda	1.19	0.0119	1.17	0.0117	7
25	Reliance	1.18	0.0118	1.15	0.0115	6
26	Cipla	1.12	0.0112	1.12	0.0112	5
27	Jaiprakash Assoc.	0.97	0.0097	1.01	0.0101	4
28	DLF	0.85	0.0085	0.85	0.0085	3
29	Reliance Comm.	0.83	0.0083	0.83	0.0083	2
30	ACC	0.68	0.0068	0.68	0.0068	1

Next we compute these values of the generalized 'useful' r-norm relative measure when R = 2 and β =0.5 in the following table:

Now,the graphically representation of the new 'useful' R-norm relative information of degree β when R=2 and β=0.5 is given in fig. 5.1.

Table 5.2: Values of'Useful' R-norm Relative Measure

P_i	q_i	u_i	$D_R^\beta(P:Q;U)$
0.1046	0.1044	30	0.000186868
0.1004	0.0993	29	-0.00036369
0.0838	0.0843	28	-0.001666033
0.0837	0.0839	27	-0.0113086
0.0641	0.0647	26	-0.000976981
0.0605	0.0613	25	0.000122931
0.0602	0.061	24	0.00205616
0.054	0.0535	23	0.004712
0.034	0.0336	22	0.00409425
0.0318	0.0314	21	0.00324745
0.0301	0.0296	20	0.00212248
0.0289	0.0285	19	0.0001693
0.0242	0.0244	18	-0.00188175
0.0229	0.023	17	-0.00108002
0.0214	0.0215	16	-0.000645739
0.0203	0.0202	15	-0.00006348
0.0194	0.0194	14	-0.000880559
0.0161	0.0161	13	-0.00107502
0.0157	0.0159	12	-0.00129549
0.0154	0.0154	11	0.000681397
0.0148	0.015	10	0.000855283
0.0129	0.0128	9	0.0414387
0.0127	0.0127	8	0.00353355
0.0119	0.0117	7	0.004735
0.0118	0.0115	6	0.00125834
0.0112	0.0112	5	-0.0085101
0.0097	0.0101	4	-0.0139873
0.0085	0.0085	3	0
0.0083	0.0083	2	0
0.0068	0.0068	1	0

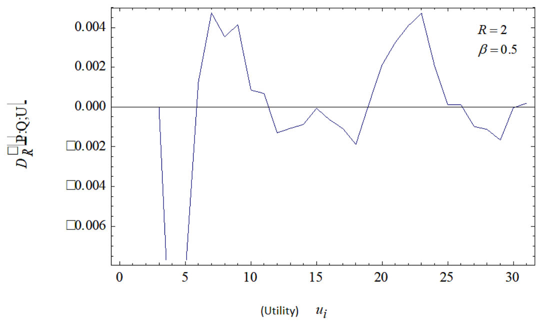

Fig.5.1: Graph of the new'useful' R-norm relative information

The amount of divergence values can be arranged for forecasting the profit maximization in a table as

Table 5.3: Data of Divergence Values

S.No.	Name of Company	Amount of Divergence
1	Bharti Airtel	0.008552
2	HDFC Bank	0.004735
3	Maruti Suzuki	0.004712
4	NTPC	0.004094
5	SBI	0.003533
6	Tata Power	0.0032471
7	Wipro	0.002122
8	Hero Honda	0.002056
9	ONGC	0.001443
10	HDFC	0.00112584
11	Tata Consult	0.000681397
12	ACC	0.000186868
13	Sterlite Industry	0.0001693
14	Reliance	0.000122931
15	ICICI Bank	0
16	Infosys Tech.	0
17	Reliance Ind.	0
18	Reliance Comm.	-0.000036369
19	Hindustan Unilever	-0.0000634824
20	Jindal Steel	-0.000645739
21	Tata Motors	-0.000880599
22	Cipla	-0.000976981
23	Tata Steel	-0.00107502
24	M&M	-0.00108002
25	BHEL	-0.00129549
26	DLF.	-0.00166033
27	Hindalco	-0.00188175
28	ITC	-0.008501
29	Jaiprakash Assoc.	-0.0113086
30	L&T	-0.0139873

Interpretation

On the basis of values of generalized divergent 'useful' R-norm relative information of degree β represented in table 5.3, we can suggest the investor to select the company having maximum divergence value for investment

Adjoint of the Generalized Information Measure and its Application

On taking Q and P after interchanging in (2.9), we get

$$D_R^{\beta}(Q:P;U)=\frac{R}{\beta-R}\left[1-\left(\frac{\sum_{i=1}^{n}u_i\,q_i^{R-\beta+1}p_i^{\beta-R}}{\sum_{i=1}^{n}u_i q_i}\right)^{\frac{1}{R-\beta+1}}\right]\quad R>0(\neq 1),\ 0<\beta\leq 1, \qquad ...(6.1)$$

Thus (6.1) is called the ad joint of (2.9). Similarly, we compute these values of the ad joint of generalized measure at R = 2, β =0.5 and representedt able (6.1) as given below:

Table 6.1: Values of adjoint of generalized measure

P_i	q_i	u_i	$D_R^{\beta}(P:Q;U)$
0.1046	0.1044	30	-0.00000089
0.1004	0.0993	29	0.000242452
0.0838	0.0843	28	0.00185256
0.0837	0.0839	27	0.00134892
0.0641	0.0647	26	0.00123747
0.0605	0.0613	25	0.000155902
0.0602	0.061	24	-0.00179842
0.054	0.0535	23	-0.00451361
0.034	0.0336	22	--0.00386599
0.0318	0.0314	21	-0.00300699
0.0301	0.0296	20	-0.00187745
0.0289	0.0285	19	0.0000332657
0.0242	0.0244	18	0.00204114
0.0229	0.023	17	0.00125102
0.0214	0.0215	16	0.000843364
0.0203	0.0202	15	0.00291878
0.0194	0.0194	14	0.00114291
0.0161	0.0161	13	0.00139457
0.0157	0.0159	12	0.00167957
0.0154	0.0154	11	-0.00276932
0.0148	0.015	10	-0.000347704
0.0129	0.0128	9	-0.0036284
0.0127	0.0127	8	-0.00288184
0.0119	0.0117	7	-0.0038725
0.0118	0.0115	6	-0.000224019
0.0112	0.0112	5	0.0092187
0.0097	0.0101	4	0.0149516
0.0085	0.0085	3	0
0.0083	0.0083	2	0
0.0068	0.0068	1	0

Considering the above table 6.1, the graph is drawn as given in the following figure 6.1:

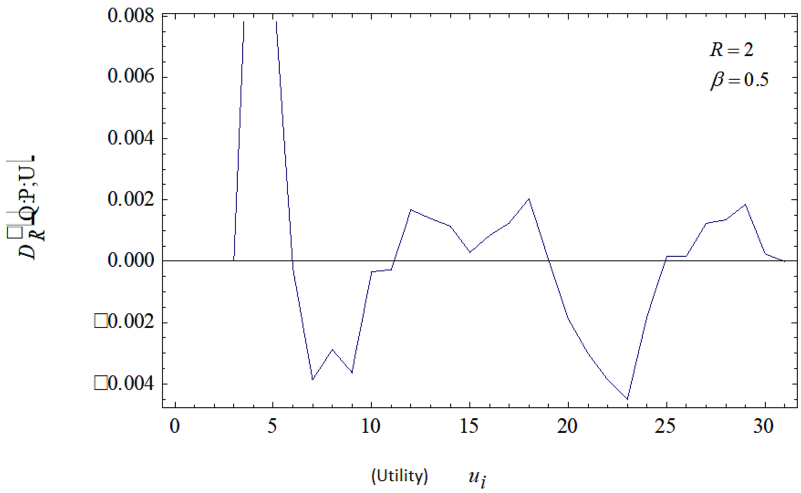

Fig. 6.1: Graph of Non-symmetric Ad joint

Table 6.2: Arrangement of Data for Forecasting Profit

Serial No.	Name of Company	Divergence values
1	ITC	0.0092187
2	Hindalco	0.00204114
3	DLF.	0.00185256
4	BHEL	0.00167957
5	Reliance Infras.	0.00155902
6	L&T	0.00149516
7	Tata Steel	0.00139457
8	Jaiprakash Assoc.	0.00134892
9	M&M	0.00125102
10	Cipla	0.00123747
11	Tata Motors	0.00114291
12	Jindal Steel	0.000843364
13	Hindustan Unilever	0.000291878
14	Reliance Comm.	0.002424432
15	Sterlite Industry	0.0000332657
16	ICICI Bank	0
17	InfosysTech.	0
18	Reliance Ind.	0
19	ACC	-0.000000898522
20	HDFC	-0.000224019
21	Tata Consult	-0.000276932
22	Bharti Airtel	-0.000347704
23	HeroHonda	-0.00179842
24	Wipro	-0.00187745
25	SBI	-0.00288184
26	Tata Power	-0.00300699
27	ONGC	-0.0036284
28	NTPC	-0.00386599
29	HDFCBank	-0.0038725
30	Maruti Suzuki	-0.00451361

The data for forecasting the profit maximization is arranged as given below:

Interpretation

The adjoint of' useful' R-norm relative information of degree in decreasing order in the table (6.1 to suggest the investor to make investment in the company of maximum divergence

Conclusion

In this paper we have defined and characterized the generalized 'useful' R-norm relative information of degree β and discussed its particular cases also. The application of this information measure has been studied. The adjoint of this measure is defined and its application in share market and decision making problems are described graphically. The 'Useful' R-norm relativei nformation measures of degree and its ad joint are defined and studied in this communication can further be generalized parametrically and applied in planning, forecasting, agriculture, etc.

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Conflict of Interest

There is no conflict of interest among the authors of this paper.

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