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# Calibration of Stratified Random Sampling with Combined Ratio Estimators

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## Abstract

This study considered modification of combined ratio type calibration estimators in stratified random sampling using calibration estimation approaches. The estimators of population mean in stratified random sampling depends on the strata estimated sample means. However, the means are sensitive to the extreme values or outliers in the sample observations of the study variables and strata sizes respectively. A new sets of calibration weights and property of the suggested combined calibration estimators of population mean in stratified sampling were derived. Empirical study through simulation was conducted to investigate the efficiency of the modified combined ratio-type calibration estimators of population mean obtained and the results revealed that the suggested estimators of population mean performed better than some existing estimators considered in the study.



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# Keywords

Calibration; Calibration Constraints; Estimators; Sampling; Stratified Random Population Mean.

### Introduction

Proper utilization of auxiliary information to obtain the efficiency of estimates of the population mean has increased in the theory of sample surveys. Many researchers have used auxiliary information in product, ratio and regression type estimators to obtain more efficient estimator under different sampling scheme. Calibration resolution is used in stratified random sampling in order to achieve optimum strata weights for precision improvement of estimates of parameters. In calibration estimation, new stratum weights are calculated to minimize a certain distance measure from the original design weights while meeting a set of auxiliary information restrictions. Deville and Sarndal<sup>1</sup> established the approach of estimate by calibration in survey sampling in 1992. The concept is to employ auxiliary data (auxiliary information) to improve a population

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statistic estimate. Following Deville and Sarndal,<sup>4</sup> Singh et al.9 was the first to extend a method of calibrating to a stratified sampling design. Many other researchers have investigated calibration estimates in survey sampling design utilizing various calibration constraints. These researchers include Singh,<sup>10</sup> Tracy et al.,<sup>11</sup> Kim et al.,<sup>6</sup> Clement and Enang,<sup>2</sup> Koyuncu and Kadilar.<sup>7</sup> In stratified sampling, Rao et al.8 suggested a multivariate calibration estimator for the population mean based on different distance measures and two auxiliary variables. In the previous studied, none have considered calibration estimation in combined ratio estimators. In this study, calibration approaches have been adopted in combined ratio estimator with aim to obtain highly efficient estimators of population mean in stratified random sampling. The presence of extreme values in the observation of the study variable have no or little effect on the other estimates.

Take a look at a finite population of elements,  $T=\{T_1, T_2, T_3,...,T_N \text{ consists of } L \text{ strata with } N_h \text{ units}$ in the *h*th stratum from which a simple random of size can be generated from the population using SRSWOR. Total Population size  $N = \sum_{h=1}^{L} N_h$ , sample size  $n = \sum_{h=1}^{L} n_h$  where  $y_{hi}$ , i=1,2,...,  $N_{hi}$  and  $x_{hi}$ , i=1,2,..., N <sub>hi</sub> of auxiliary variable x and study variable y. Let  $W_h = N_h / N$  be the weights of the strata,  $\bar{y}_h = n^2 \sum_{h=1}^{L} y_{hi}$  the sample mean and  $\bar{y}_h = N^2 \sum_{h=1}^{L} y_{hi}$  population mean for the study variable.

#### Literature Review

According to Cochran,<sup>3</sup> with stratified sampling, the classic estimator of population mean is given as:

$$\overline{y}_{st} = \sum_{h=1}^{L} W_h \overline{y}_h \qquad \dots (2.1)$$

$$V(\bar{y}_{st}) = \sum_{h=1}^{L} W_h^2 \left( \frac{1 - f_h}{n_h} \right) s_{hy}^2 \qquad \dots (2.2)$$

where  $s_{hy}^2 = (n_h - 1)^{-1} \sum_{h=1}^{n} (y_{hi} - \overline{y}_h)^2$ 

Hansen *et al.*<sup>5</sup> suggested a combined ratio estimator as

$$\overline{y}_{zt}^{RC} = \frac{\sum_{i=1}^{n_h} W_h \overline{y}_h}{\sum_{i=1}^{n_h} W_h \overline{x}_h} \overline{X} \qquad \dots (2.3)$$

The combined ratio estimator's variance is given as follows:

$$V\left(\overline{y}_{st}^{RC}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2 + R^2 S_{xh}^2 - 2RS_{yxh}\right) \qquad \dots (2.4)$$

where  $R = \frac{r}{\overline{X}}$ 

Singh *et al.*<sup>9</sup> presented the calibration approach for the combined general regression estimator (GREG) for the population mean given by

$$\overline{y}_{st(SH)} = \sum_{h=1}^{L} \Omega_h^{SH} \overline{y}_h \qquad \dots (2.5)$$

By minimizing the Chi-Square distance measure, Singh *et al.*<sup>9</sup> were able to get new calibration weights.

$$Z_{1} = \sum_{h=1}^{L} \left( \Omega_{h}^{SH} - W_{h} \right)^{2} / Q_{h} W_{h} \qquad \dots (2.6)$$

subject to the constraint

$$\sum_{h=1}^{L} \Omega_{h}^{SH} \overline{x}_{_{1h}} = \overline{X} \qquad \dots (2.7)$$

The calibrated weights and the estimator are obtained as show in (2.6) and (2.7) respectively.

$$\Omega_{h}^{SH} = W_{h} + \left(\sum_{h=1}^{L} W_{h} Q_{h} \overline{x}_{h} / \sum_{h=1}^{L} W_{h} Q_{h} \overline{x}_{h}^{2}\right) \left(\overline{X} - \sum_{h=1}^{L} W_{h} \overline{x}_{h}\right)$$
...(2.8)

As a result, the modified calibrated estimator of Singh *et al.*<sup>9</sup> is

$$\overline{y}_{zt(SH)} = \sum_{h=1}^{L} W_h \overline{y}_h + \left( \sum_{h=1}^{L} W_h Q_h \overline{x}_h \overline{y}_h / \sum_{h=1}^{L} W_h Q_h \overline{x}_h^2 \right) \left( \overline{X} - \sum_{h=1}^{L} W_h \overline{x}_h \right)$$
...(2.9)

The calibrated estimator's  $\overline{\mathcal{Y}}_{\textit{st(SH)}}$  estimated variance is given by

$$V\hat{a}r\left(\overline{y}_{st(SH)}\right) = \sum_{h=1}^{L} \Omega_h^{SH^2} \gamma_h s_{eh}^2 \qquad \dots (2.10)$$

where  $s_{ab}^2 = (n_h - 1)^{-1} \sum_{h=1}^{m} e_{h}^2$  is the *h*<sup>th</sup> strata sample mean square and  $e_{hi} = (y_{hi} - \overline{y}_h) - b(x_{hi} - \overline{x}_h)$  with  $b = \sum_{h=1}^{L} W_h Q_h \overline{x}_h \overline{y}_h / \sum_{h=1}^{L} W_h Q_h \overline{x}_h^2$ 

Singh<sup>10</sup> introduced new calibration equations to a population mean calibration estimator in stratified sampling. Under stratified sampling, the Singh<sup>3</sup> calibration estimator of the population mean  $\bar{Y}$  is given by

$$\overline{y}_{st(S)} = \sum_{h=1}^{L} \Omega_h^S \overline{y}_h \qquad \dots (2.11)$$

where  $\Omega_{h}^{s}$  is calibrated weight which is chosen so that the Chi-Square distance mean sum is

$$Z_{2} = \sum_{h=1}^{L} \left( \Omega_{h}^{S} - W_{h} \right)^{2} / \mathcal{Q}_{h} W_{h} \qquad \dots (2.12)$$

is subject to the following constraints as a minimum

$$\sum_{h=1}^{L} \Omega_h^S = 1 \qquad \dots (2.13)$$

$$\sum_{h=1}^{L} \Omega_h^S \overline{x}_h = \overline{X} \qquad \dots (2.14)$$

The calibrated weight and the estimator (2.11) are obtained as show in (2.15) and (2.16) respectively

$$\Omega_{h}^{\sharp} = W_{h} + \frac{\left(W_{h}Q_{h}\overline{x}_{h}\right)\left(\sum_{h=1}^{L}W_{h}Q_{h}\right) - \left(W_{h}Q_{h}\right)\left(\sum_{h=1}^{L}W_{h}Q_{h}\overline{x}_{h}\right)}{\left(\sum_{h=1}^{L}W_{h}Q_{h}\right)\left(\sum_{h=1}^{L}W_{h}Q_{h}\overline{x}_{h}^{2}\right) - \left(\sum_{h=1}^{L}W_{h}Q_{h}\overline{x}_{h}\right)^{2}} \left(\overline{X} - \sum_{h=1}^{L}W_{h}\overline{x}_{h}\right)$$
...(2.15)

$$\overline{y}_{st(S)} = \sum_{h=1}^{L} W_h \overline{y}_h + \hat{\beta}_{(s)} \left( \overline{X} - \sum_{h=1}^{L} W_h \overline{x}_h \right) \qquad \dots (2.16)$$

where

$$\hat{\beta}_{(S)} = \frac{\left(\sum_{h=1}^{L} W_h \mathcal{Q}_h \overline{x}_h \overline{y}_h\right) \left(\sum_{h=1}^{L} W_h \mathcal{Q}_h\right) - \left(\sum_{h=1}^{L} W_h \mathcal{Q}_h \overline{y}_h\right) \left(\sum_{h=1}^{L} W_h \mathcal{Q}_h \overline{x}_h\right)}{\left(\sum_{h=1}^{L} W_h \mathcal{Q}_h\right) \left(\sum_{h=1}^{L} W_h \mathcal{Q}_h \overline{x}_h^2\right) - \left(\sum_{h=1}^{L} W_h \mathcal{Q}_h \overline{x}_h\right)^2}$$

The estimated variance of Singh<sup>10</sup> calibrated estimator is given by

$$\mathcal{V}\hat{a}r\left(\overline{y}_{st(S)}\right) = \sum_{h=1}^{L} \Omega_{h}^{S^{2}} \gamma_{h} S_{eh}^{2} \qquad \dots (2.17)$$

where  $s_{eh}^2 = (n_h - 1)^{-1} \sum_{i=1}^{m} e_{hi}^2$  is the mean square of the hth stratum sample while .

$$e_{hi} = \left(y_{hi} - \overline{y}_{h}\right) - \hat{\beta}_{(s)}\left(x_{hi} - \overline{x}_{h}\right)$$

Alam *et al.*<sup>1</sup> developed a calibration estimator for estimating population mean under stratified random sampling using a distance function as follows

$$\overline{y}_{st}^{\mathcal{A}} = \sum_{h=1}^{L} \Omega_h^{\mathcal{A}} \overline{y}_h \qquad \dots (2.18)$$

where  $\Omega_h^{\scriptscriptstyle A}$  are the calibration weights which are chosen with the distance function

$$Z_{3} = \frac{1}{2} \sum_{h=1}^{L} \frac{\left(\Omega_{h}^{A} - W_{h}\right)^{2}}{W_{h}Q_{h}} + \sum_{h=1}^{L} \sum_{h\neq h'=1}^{L} \left(\Omega_{h}^{A} - W_{h}\right) \left(\Omega_{h'}^{A} - W_{h'}\right) \dots (2.19)$$

where  $\Omega_{h1}$  is the h<sup>1 th</sup> stratum weight which minimum subject to the calibration constraints

$$\sum_{h=1}^{L} \Omega_h^A \overline{x}_h = \overline{X} \qquad \dots (2.20)$$

Minimization of (2.19) subject to the calibration constraint given in (2.20), the calibration weights are given by

$$\Omega_h^A = W_h + \left(\frac{\bar{X} - \sum_{h=1}^L W_h \bar{x}_h}{\sum_{h=1}^L \left(\frac{W_h Q_h \bar{x}_h^2}{1 - W_h Q_h}\right)}\right) \left(\frac{W_h Q_h \bar{x}_h}{1 - W_h Q_h}\right) \dots (2.21)$$

And the calibrated estimator is

$$\bar{y}_{st}^{A} = \sum_{h=1}^{L} W_{h} \bar{y}_{h} + \hat{\beta}_{a} \left( \bar{X} - \sum_{h=1}^{L} W_{h} \bar{x}_{h} \right) \qquad \dots (2.22)$$
$$\hat{\beta}_{a} = \frac{\sum_{h=1}^{L} \left( \frac{W_{h} Q_{h} \bar{x}_{h} \bar{y}_{h}}{1 - W_{h} Q_{h}} \right)}{\sum_{h=1}^{L} \left( \frac{W_{h} Q_{h} \bar{x}_{h}^{2}}{1 - W_{h} Q_{h}} \right)} \qquad \dots (2.23)$$

The estimated variance of Alam *et al.*<sup>1</sup> calibration estimator is given by

$$V\left(\overline{y}_{st}^{A}\right) = \sum_{h=1}^{L} \Omega_{h}^{A^{2}} \gamma_{h} s_{eh}^{2} \qquad \dots (2.24)$$

#### **Proposed Estimator**

In stratified random sampling, the conventional Combined Ratio Estimator is given in (2.3) can be written as

$$\overline{y}_{st}^{RC} = \sum_{h=1}^{L} W_h^* \overline{y}_h \qquad \dots (2.25)$$

where  $W_h^* = W_h \overline{X} / \sum_{h=1}^L W_h \overline{x}_h$ 

Motivated by Alam *et al.*, <sup>1</sup> calibrated combined ratio estimator denoted by  $\overline{y}_{\rm st(M)}^{\rm RC}$  is modified as

$$\overline{y}_{st(M)}^{RC} = \sum_{h=1}^{L} \Omega_h \overline{y}_h \qquad \dots (2.26)$$

where  $\Omega_{\!_{h}}$  denotes the new calibrated weights that minimize the Chi-square distance.

$$Z_{4}^{*} = \frac{1}{2} \sum_{h=1}^{L} \frac{\left(\Omega_{h}^{4} - W_{h}^{*}\right)^{2}}{W_{h}^{*} Q_{h}} + \sum_{h=1}^{L} \sum_{h\neq h'=1}^{L} \left(\Omega_{h}^{4} - W_{h}^{*}\right) \left(\Omega_{h'}^{4} - W_{h'}^{*}\right) \dots (2.27)$$

Subject to calibration constraint given by

$$\sum_{h=1}^{L} \Omega_{h} \overline{x}_{h} = \sum_{h=1}^{L} W_{h}^{*} \overline{X}_{h} \qquad ...(2.28)$$

The Lagrange multipliers technique is employed to compute new calibrated weights( $\Omega_h$ ), and the following results are obtained

$$L = \frac{1}{2} \sum_{h=1}^{L} \frac{\left(\Omega_{h}^{4} - W_{h}^{*}\right)^{2}}{W_{h}^{*} Q_{h}} + \sum_{h=1}^{L} \sum_{h=l}^{L} \left(\Omega_{h}^{4} - W_{h}^{*}\right) \left(\Omega_{ll}^{4} - W_{ll}^{*}\right) - \lambda \left(\sum_{h=1}^{L} \Omega_{h} \bar{x}_{h} - \sum_{h=1}^{L} W_{h}^{*} \bar{X}_{h}\right)$$
...(2.29)

Differentiating (2.29) partially in relation to  $\Omega_h$ , and  $\lambda$ , equal to zero

$$\Omega_{h} = W_{h}^{*} + \lambda \left( \frac{W_{h}^{*} Q_{h} \overline{x}_{h}}{1 - W_{h}^{*} Q_{h}} \right) \qquad \dots (2.30)$$

$$\sum_{h=1}^{L} \Omega_{h} \overline{x}_{h} - \sum_{h=1}^{L} W_{h}^{*} \overline{X}_{h} = 0 \qquad ...(2.31)$$

Substitute (2.30) in (2.31), the results are obtained as

$$\lambda \sum_{h=1}^{L} \left( \frac{W_h^* Q_h \overline{x}_h^2}{1 - W_h^* Q_h} \right) = \sum_{h=1}^{L} W_h^* \overline{X}_h - \sum_{h=1}^{L} W_h^* \overline{x}_h \quad \dots (2.32)$$

Make  $\lambda$  the subject of formula, obtained as:

$$\lambda = \frac{\sum_{h=1}^{L} W_{h}^{*} \bar{X}_{h} - \sum_{h=1}^{L} W_{h}^{*} \bar{x}_{h}}{\sum_{h=1}^{L} \left( \frac{W_{h}^{*} Q_{h} \bar{x}_{h}^{2}}{1 - W_{h}^{*} A_{h} Q_{h}} \right)} \qquad \dots (2.33)$$

On substituting (2.33) in (2.30) the calibrated weights can be written as

$$\Omega_{h} = W_{h}^{*} + \left(\frac{\sum_{h=1}^{L} W_{h}^{*} \bar{X}_{h} - \sum_{h=1}^{L} W_{h}^{*} \bar{x}_{h}}{\sum_{h=1}^{L} \left(\frac{W_{h}^{*} Q_{h} \bar{x}_{h}^{2}}{1 - W_{h}^{*} p_{h}}\right)}\right) \left(\frac{W_{h}^{*} Q_{h} \bar{x}_{h}}{1 - W_{h}^{*} Q_{h}}\right) \dots (2.34)$$

Substituting (2.34) in (2.26), obtain the new combined calibration estimator  $(\overline{y}_{st(M)}^{RC})$  as

$$\overline{y}_{st(M)}^{RC} = \sum_{h=1}^{L} W_h^* \overline{y} + \hat{\beta} \sum_{h=1}^{L} W_h^* \left( \overline{X}_h - \overline{x}_h \right) \qquad \dots (2.35)$$

Substituting  $W_h^* = W_h \overline{X} / \sum_{h=1}^{L} W_h \overline{x}_h$  in (2.35), gives

$$\overline{y}_{\pi(M)}^{RC} = \overline{X}\left(\sum_{h=1}^{L} W_{h} \overline{y}\right) \left(\sum_{h=1}^{L} W_{h} \overline{x}_{h}\right)^{-1} + \hat{\beta} \overline{X}\left(\sum_{h=1}^{L} W_{h}\right) \left(\sum_{h=1}^{L} W_{h} \overline{x}_{h}\right)^{-1} \left(\overline{X}_{h} - \overline{x}_{h}\right) \qquad \dots (2.36)$$

where 
$$\hat{\beta} = \frac{\sum_{h=1}^{L} \left( \frac{W_h Q_h \overline{x}_h \overline{y}_h}{1 - W_h Q_h} \right)}{\sum_{h=1}^{L} \left( \frac{W_h Q_h \overline{x}_h^2}{1 - W_h Q_h} \right)}$$

Setting  $Q_h = 1$ , and  $Q_h = \overline{x}_h^{-1}$  we have the following new set of calibration combined ratio estimators respectively

$$\bar{y}_{\mathfrak{A}(M1)}^{\mathcal{R}} = \bar{X} \left( \sum_{h=1}^{L} W_{h} \bar{y} \right) \left( \sum_{h=1}^{L} W_{h} \bar{x}_{h} \right)^{-1} + \hat{\beta}_{1} \bar{X} \left( \sum_{h=1}^{L} W_{h} \right) \left( \sum_{h=1}^{L} W_{h} \bar{x}_{h} \right)^{-1} \left( \bar{X}_{h} - \bar{x}_{h} \right) \dots (2.37)$$

$$\overline{y}_{d(M2)}^{RC} = \overline{X} \left( \sum_{h=1}^{L} W_h \overline{y} \right) \left( \sum_{h=1}^{L} W_h \overline{x}_h \right)^{-1} + \hat{\beta}_2 \overline{X} \left( \sum_{h=1}^{L} W_h \right) \left( \sum_{h=1}^{L} W_h \overline{x}_h \right)^{-1} \left( \overline{X}_h - \overline{x}_h \right) \dots (2.38)$$

where 
$$\hat{\beta}_1 = \frac{\sum\limits_{h=1}^{L} \left( \frac{\overline{W}_h \overline{x}_h \overline{y}_h}{1 - \overline{W}_h} \right)}{\sum\limits_{h=1}^{L} \left( \frac{\overline{W}_h \overline{x}_h^2}{1 - \overline{W}_h} \right)}$$
, and  $\hat{\beta}_2 = \frac{\sum\limits_{h=1}^{L} \left( \frac{\overline{W}_h \overline{y}_h}{1 - \overline{W}_h \overline{x}_h^{-1}} \right)}{\sum\limits_{h=1}^{L} \left( \frac{\overline{W}_h \overline{x}_h}{1 - \overline{W}_h \overline{x}_h^{-1}} \right)}$ 

The suggested estimator's estimated variance

$$\hat{Var}\left(\overline{y}_{st(M)}^{RC}\right) = \sum_{i=1}^{L} \Omega_h^2 \gamma_h s_{eh}^2 \qquad \dots (2.39)$$

where  $s_{eh}^2 = \left(n_h - 1\right)^{-1} \sum_{i=1}^{n_h} e_{ih}^2$  is the hth strata sample

mean square and  $e_{\scriptscriptstyle th} = \left( \, y_{\scriptscriptstyle ht} - \overline{y}_{\scriptscriptstyle h} \, \right) - \hat{eta} \left( \, x_{\scriptscriptstyle ht} - \overline{x}_{\scriptscriptstyle h} \, \right)$  .

# **Empirical Study Using Simulation**

In this section, a simulation research was carried out to see if the proposed estimators were better than the other estimators evaluated in the study. For this investigation, 1000-unit data was generated. Using the function defined in Table 1, populations were stratified into three non-overlapping heterogeneous groups of 200, 300, and 500. Method SRSWOR was used to randomly choose samples of sizes 20, 30, and 50 from each stratum 10,000 times. The precision (PRE) of the estimators under consideration was calculated using (2.40)

$$PRE(\hat{\theta}_{i}) = (\operatorname{var}(\overline{y}_{st}) / \operatorname{var}(\theta_{i})) 100 \qquad \dots (2.40)$$

where 
$$\operatorname{var}(\theta_l) = \frac{1}{10000} \sum_{j=1}^{10000} (\theta_l - \overline{Y})^2, \ \theta_l = \overline{y}_{st}^{RC}, \ \overline{y}_{st(S)}, \ \overline{y}_{st(M)}^{RC}.$$

Populations	Auxiliary variable x	Study variable <i>y</i>
I	$x_h \approx \operatorname{chisq}(\theta_h), \theta_1 = 5, \theta_2 = 6,$ $\theta_3 = 4, h = 1, 2, 3$	$y_{hi} = \alpha_h x_{hi}^2 + \xi_{hi}, \ \alpha_{1h} = E(x_h),$ $\alpha = 0.5, \ \xi_h \approx N(0,1), h = 1, 2, 3$
II	$\begin{aligned} x_h &\approx gamma\big(\theta_h, \eta_h\big), \theta_1 = 3, \eta_1 = 2, \\ \theta_2 &= 3, \eta_2 = 1, \theta_3 = 3, \eta_3 = 3, \end{aligned}$	

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Estimator	$y_{hi}=lpha_h x_{hi}^2+oldsymbol{\xi}_{hi}$		
	$Q_h=1$	$Q_h = \overline{x}_h^{-1}$	
$\overline{\mathcal{Y}}_{st}$	100	100	
Combined ratio $\overline{y}_{st}^{RC}$	276.4634	276.4634	
Alam <i>et al. et al.</i> <sup>1</sup> $\overline{y}_{st}^{A}$	350.0407	361.6427	
Suggested Estimator $\overline{\mathcal{Y}}_{st(M)}^{RC}$	483.9979	459.3958	
Estimator	$y_{hi} = lpha_h x_{hi}^3 + \xi_{hi}$		
$\overline{\mathcal{Y}}_{st}$	100	100	
Combined ratio $\overline{y}_{st}^{RC}$	163.2217	163.2217	

Alam <i>et al.</i> <sup>1</sup> $\overline{\mathcal{Y}}_{st}^{A}$	184.4712	185.8651
Suggested Estimator $\overline{\mathcal{Y}}_{\textit{st}(M)}^{\textit{RC}}$	247.3609	246.3516
Estimator	$y_{hi} = \alpha_h x_{hi}^4 + \xi_{hi}$	
$\overline{\mathcal{Y}}_{st}$	100	100
Combined ratio $\overline{y}_{st}^{RC}$	131.9602	131.9602
Alam <i>et al.</i> <sup>1</sup> $\overline{y}_{st}^{A}$	141.1144	140.7026
Suggested Estimator $\overline{\mathcal{Y}}_{st(M)}^{RC}$	165.7013	165.0921

Table 3: PREs of Some Existing and Suggested Estimators Using Population II

Estimator	$y_{hi} = lpha_h x_{hi}^2 + \xi_{hi}$	
	<i>Q</i> <sub><i>h</i></sub> = 1	$Q_h = \overline{x}_h^{-1}$
$\overline{\mathcal{Y}}_{st}$	100	100
Combined ratio $\overline{y}_{st}^{RC}$	197.4018	197.4018
Alam <i>et al.</i> <sup>1</sup> $\overline{y}_{st}^{A}$	272.0550	266.2928
Suggested Estimator $\overline{y}_{st(M)}^{RC}$	355.2338	345.9467
Estimator $y_{hi} = lpha_h x_{hi}^3 + \xi_{hi}$		ii
$\overline{\mathcal{Y}}_{st}$	100	100
Combined ratio $\overline{y}_{st}^{RC}$	143.1531	143.1531
Alam et al. <sup>1</sup> $\overline{y}_{st}^{A}$	182.1500	177.3475
Suggested Estimator $\overline{y}_{st(M)}^{RC}$	233.6707	229.5379
Estimator	$y_{hi} = lpha_h x_{hi}^4 + \xi_{hi}$	
$\overline{\mathcal{Y}}_{st}$	100	100
Combined ratio $\overline{y}_{st}^{RC}$	124.2426	124.2426
Alam <i>et al.</i> <sup>1</sup> $\overline{y}_{st}^{A}$	144.5459	140.7251
Suggested Estimator $\overline{y}_{st(M)}^{RC}$	166.9500	163.7467

Tables 2 and 3 shown the PREs of some existing and suggested estimators considered in this study using data generated by populations I and II respectively. From the results obtained, it revealed that the suggested estimators outperformed the existing estimators considered in the study.

#### Conclusion

From the results obtained, the empirical study revealed that on the efficiency of the suggested calibration estimators versus the study's current related estimators, the suggested estimators have higher PREs compared to some existing calibration estimators in the numerical analysis. The suggested estimators outperformed other calibration estimators because the suggested estimators demonstrated high level of efficiency over other estimators. Hence, the suggested estimators are closers to the true values of the population mean compared to other existing calibration estimators in which the suggested estimators have more chances of producing estimates that are closer to the population mean's true value.

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#### **Conflict of Interest**

The authors declare no conflict of interest.

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