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Improved Modified Classes of Regression Type Estimators of Finite Population Mean in the Presence of Auxiliary Attribute

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Abstract

In this research, estimators are suggested toimprove modified classes of regression type estimators of finite population mean. The essence of proposing the estimators is as a result of the assumption that there may be weak relationship between study variable and auxiliary attribute. Properties (Biases and MSEs) of the proposed estimators are procured using Taylor series method. The efficiency conditions under which the proposed estimators are better than other related ones are established. Empirical findings are incentive and the results shown that the proposed estimators are more proficient compare to the existing estimators considered in the study.

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Auxiliary attribute; Bias; Efficiency; Ratio Estimator; Study variable.

Introduction

In sampling theory, auxiliary informationis used to increase the precision of estimate of an estimator whenthere is correlation between response variable and auxiliary variable. Authors have suggested different estimators note varying renowned parameters of an auxiliary information. In probability sampling, it is well confirmed that auxiliary variable may be qualitative form, such variable is term auxiliary attribute. Many authors in literature have suggested estimators using auxiliary variable, they include Cochran[1] who developed the ratio estimator to investigate problem of estimation of the population mean when auxiliary variable is present. Other researchers that developed estimators using auxiliary information include. Audu *et al.*,[1] Tailor *et al.*,[11] Singh,[6,7] Kadilarand Cingi,[3] Khoshnevisan *et al.*,[4] Perri,[5] Yunusa *et al.*,[12] Singh and Kumar.[8]

When the auxiliary information are qualitative in nature, that is, auxiliary information in the form of attribute, such as colour of hair of individuals and their weight can be regarded as auxiliary attribute

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and study variable, sex and height of women in a locality may be regarded as auxiliary attribute and study variable etc. Several authors have developed estimators in this direction likeSingh *et al.*,[9,10] Zaman[13] and Zaman and Kadilar.[14]

Currently in this research, we have intended efficient regression type estimators of finite population mean, that gives precise estimate for the size of finite population mean in the presence of auxiliary attribute when the bi-serial correlation between study variable and auxiliary attribute is weak.

Materials and Methods

Sample mean $\bar{y}_m$ of simple random sampling is given as

$$\bar{y}_m = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad ...(1)$$

Bias and variance of $\bar{y}_m$ is given by

$$Bias(\bar{y}_m) = 0 \qquad ...(2)$$

$$Var(\bar{y}_m) = \gamma \bar{Y}^2 C_y^2 \qquad ...(3)$$

where $\gamma = \frac{1-f}{n}$, $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$, $C_y = \frac{S_y}{\bar{Y}}$

Zaman and Kadilar[14] class of exponential ratio type estimators in the presence of auxiliary attributeas.

$$\bar{y}_{zk} = \bar{y}\exp\left(\frac{(kP+l)-(kp+l)}{(kP+l)+(kp+l)}\right) \qquad ...(4)$$

Bias and mean square error of the estimator $\bar{y}_{zk}$ are given by

$$Bias(\bar{y}_{zk}) = \gamma \bar{Y}\left(\theta_i^2 C_\phi^2 + \theta_i \rho_{y\phi} C_y C_\phi\right) \qquad ...(5)$$

$$MSE(\bar{y}_{zk}) = \gamma \bar{Y}^2\left(C_y^2 + \theta_i^2 C_\phi^2 - 2\theta_i \rho_{y\phi} C_y C_\phi\right) \qquad ...(6)$$

Zaman,[13] an improved class of estimator for the estimation of population mean as

$$\bar{y}_{zi} = \bar{y}\left(\frac{p}{P}\right)^{\alpha}\exp\left(\frac{(kP+l)-(kp+l)}{(kP+l)+(kp+l)}\right) \qquad ...(7)$$

The MSE of the estimator is

$$MSE(\bar{y}_{zi}) = \gamma \bar{Y}^2\left(\begin{array}{l} C_y^2 + \alpha^2 C_\phi^2 + \theta_i^2 C_\phi^2 - 2\alpha\theta_i C_\phi^2 + 2\alpha\rho_{y\phi} C_y C_\phi \\ -2\theta_i C_y C_\phi \end{array}\right) \qquad ...(8)$$

Where $\alpha = \frac{\theta_i C_\phi - \rho_{y\phi} C_y}{C_\phi}$, $i = 0,1,2,...,9$.

$$MSE(\bar{y}_{zi})_{\min} = \gamma \bar{Y}^2 C_y^2\left(1-\rho_{y\phi}^2\right) \qquad ...(9)$$

Audu *et al.*[1] modified class of estimators for the population mean of the study variable in the presence of auxiliary attributeas

$$t_{pi} = \left(\bar{y} + b_\phi(P-p)\right)\exp\left(\frac{(kP+l)-(kp+l)}{(kP+l)+(kp+l)}\right) \quad i = 0,1,2,....,9 \qquad ...(10)$$

$$t_{qi} = \frac{\left(\bar{y} + b_\phi(P-p)\right)(kP+l)}{(kp^*+l)}\exp\left(\frac{(kp^*+l)-(kP+l)}{(kp^*+l)+(kP+l)}\right) \quad i = 0,1,2,..,9 \qquad ...(11)$$

bias and mean square error of the estimator t_{pi} and t_{qi} are given by

$$Bias(t_{pi}) = \gamma\left[\left(\frac{3w_2^2\bar{Y}}{8} + b_\phi w_2 P\right)C_\phi^2 - \bar{Y}w_2\rho_{y\phi}C_y C_\phi\right] \quad i = 1,2,...,9 \qquad ...(12)$$

$$Bias(t_{qi}) = -\gamma\bar{Y}\left(g^2 w_1 w_2 - \frac{g^2 w_2^2}{2}\right)C_\phi^2 \quad i = 0,1,2,...,9 \qquad ...(13)$$

$$MSE(t_{pi}) = \gamma\bar{Y}^2\left(C_y^2 + \left(w_1 + \frac{b_\phi P}{\bar{Y}}\right)^2 C_\phi^2 - 2\left(w_1 + \frac{b_\phi P}{\bar{Y}}\right)\rho_{y\phi}C_y C_\phi\right) \quad i = 1,2,..,9 \qquad ...(14)$$

$$MSE(t_{qi}) = \gamma\left(\bar{Y}^2 C_y^2 + b_\phi^2 P^2 C_\phi^2 - 2\bar{Y}Pb_\phi\rho_{y\phi}C_y C_\phi\right) \quad i = 0,1,2,...,9 \qquad ...(15)$$

Where, $w_1 = \frac{kP}{2(kP+l)}$, $w_2 = \frac{kP}{(kP+l)}$, $g = \frac{n}{N-n}$, $p^* = \frac{NP-np}{N-n}$, $b_\phi = \frac{\rho_{y\phi}\bar{Y}C_y}{C_\phi}$.

Suggested Estimators

By exploiting the idea of Audu *et al.*[1] and other estimators in literature, finite population mean based on the presence of auxiliary attribute for estimation of population mean of study variable are proposed

$$T_{ri}=\left[\bar{y}_h+a_i\left(P-p\right)+b_i\bar{y}\right]\exp\left(\frac{\left(kP+l\right)-\left(kp+l\right)}{\left(kP+l\right)+\left(kp+l\right)}\right) \quad ...(16)$$

$$T_{qi}=\frac{\left[\bar{y}_h+u_i\left(P-p\right)+v_i\bar{y}\right]\left(kP+l\right)}{\left(kp^*+l\right)}\exp\left(\frac{\left(kp^*+l\right)-\left(kP+l\right)}{\left(kp^*+l\right)+\left(kP+l\right)}\right) \quad ...(17)$$

Where $\bar{y}_h=\frac{\bar{y}}{2}\left(\exp\left(\frac{P-p}{P+p}\right)+\exp\left(\frac{p-P}{P+p}\right)\right)$,

a_i, b_i, u_i, and v_i are invariable to be determined, i=1,2,...,10. The suggested estimators will be defined if and only if $a_i\neq 0$, $b_i\neq 0$, $u_i\neq 0$ $v_i\neq 0$, and $\bar{y}_h\neq 0$. The estimator was obtained by incorporating unknowns into the estimators and taken the sample mean in the estimators of Audu *et al.*[1] as the average of the exponential ratio and product type estimators.

Table 1: Members of the proposed estimators T_{ri}.

Estimators		Values of k and l	
		K	*l*
1	$T_{r1}=\left[\bar{y}_h+a_1\left(P-p\right)+b_1\bar{y}\right]\exp\left(\frac{P-p}{P+p}\right)$	1	0
2	$T_{r2}=\left[\bar{y}_h+a_2\left(P-p\right)+b_2\bar{y}\right]\exp\left(\frac{P-p}{P-p+2\beta_{2(\phi)}}\right)$	1	$\beta_{2(\phi)}$
3	$T_{r3}=\left[\bar{y}_h+a_3\left(P-p\right)+b_3\bar{y}\right]\exp\left(\frac{P-p}{P-p+2C_\phi}\right)$	1	C_ϕ
4	$T_{r4}=\left[\bar{y}_h+a_4\left(P-p\right)+b_4\bar{y}\right]\exp\left(\frac{P-p}{P-p+2\rho_{y\phi}}\right)$	1	$\rho_{y\phi}$
5	$T_{r5}=\left[\bar{y}_h+a_5\left(P-p\right)+b_5\bar{y}\right]\exp\left(\frac{\beta_{2(\phi)}\left(P-p\right)}{\beta_{2(\phi)}\left(P+p\right)+2C_\phi}\right)$	$\beta_{2(\phi)}$	C_ϕ
6	$T_{r6}=\left[\bar{y}_h+a_6\left(P-p\right)+b_6\bar{y}\right]\exp\left(\frac{C_\phi\left(P-p\right)}{C_\phi\left(P-p\right)+2\beta_{2(\phi)}}\right)$	C_ϕ	$\beta_{2(\phi)}$
7	$T_{r7}=\left[\bar{y}_h+a_7\left(P-p\right)+b_7\bar{y}\right]\exp\left(\frac{C_\phi\left(P-p\right)}{C_\phi\left(P-p\right)+2\rho_{y\phi}}\right)$	C_ϕ	$\rho_{y\phi}$
8	$T_{r8}=\left[\bar{y}_h+a_8\left(P-p\right)+b_8\bar{y}\right]\exp\left(\frac{\rho_{y\phi}\left(P-p\right)}{\rho_{y\phi}\left(P-p\right)+2C_\phi}\right)$	$\rho_{y\phi}$	C_ϕ
9	$T_{r9}=\left[\bar{y}_h+a_9\left(P-p\right)+b_9\bar{y}\right]\exp\left(\frac{\beta_{2(\phi)}\left(P-p\right)}{\beta_{2(\phi)}\left(P-p\right)+2\rho_{y\phi}}\right)$	$\beta_{2(\phi)}$	$\rho_{y\phi}$
10	$T_{r10}=\left[\bar{y}_h+a_{10}\left(P-p\right)+b_{10}\bar{y}\right]\exp\left(\frac{\rho_{y\phi}\left(P-p\right)}{\rho_{y\phi}\left(P-p\right)+2\beta_{2(\phi)}}\right)$	$\rho_{y\phi}$	$\beta_{2(\phi)}$

To obtain the biases and MSEs of T_{ri} and T_{qi}, the following error terms are defined as $e_0=\frac{\bar{y}-\bar{Y}}{\bar{Y}}$ and such that and $e_1=\frac{p-P}{P}$ such that $\bar{y}=\bar{Y}(1+e_0)$ and $p=P(1+e_1)$

$$\left.\begin{array}{l} E(e_h)=0, h=0,1., E(e_0^2)=\lambda C_y^2, \\ E(e_1^2)=\lambda C_y^2, E(e_0e_1)=\lambda\rho_{y\phi}C_yC_\phi \end{array}\right\} \quad ...(18)$$

Expressing (16) and (17) in terms of error terms, we have

$$T_{ri}=\left[\bar{Y}\left(1+\frac{e_1^2}{8}+e_0\right)-a_iPe_1+b_i\bar{Y}(1+e_0)\right]\exp\left(\frac{-kPe_1}{2(kP+l)+kPe_1}\right) \quad ...(19)$$

$$T_{qi}=\frac{\left[\bar{Y}\left(1+\frac{e_1^2}{8}+e_0\right)-u_iPe_1+v_i\bar{Y}(1+e_0)\right](kP+l)}{(kP+l-kPge_1)}\exp\left(\frac{-kPge_1}{2(kP+l)-kPge_1}\right) \quad ...(20)$$

Simplify (19) and (20),then obtained

$$T_{ri}-\bar{Y}=\bar{Y}\left[\begin{array}{l}\left(e_0-\frac{\theta_ie_1}{2}+\frac{(3\theta_i^2+1)e_1^2}{8}-\frac{\theta_ie_0e_1}{2}\right)-a_i\frac{P}{\bar{Y}}\left(e_1-\frac{\theta_ie_1^2}{2}\right)\\ +b_i\left(1+e_0-\frac{\theta_ie_1}{2}+\frac{3\theta_i^2e_1^2}{8}-\frac{\theta_ie_0e_1}{2}\right)\end{array}\right] \quad ...(21)$$

$$T_{qi}-\bar{Y}=\bar{Y}\left[\begin{array}{l}\left(e_0+\frac{g\theta_ie_1}{2}+\frac{(3g^2\theta_i^2+1)e_1^2}{2}+\frac{g\theta_ie_0e_1}{2}\right)-u_i\frac{P}{\bar{Y}}\left(e_1+\frac{g\theta_ie_1^2}{2}\right)\\ +v_i\left(1+e_0+\frac{g\theta_ie_1}{2}+\frac{3g^2\theta_i^2e_1^2}{8}+\frac{g\theta_ie_0e_1}{2}\right)\end{array}\right] \quad ...(22)$$

Where, $\theta_i=kP/(kP+l), i=1,2,.....,10$.

Taking expectation of (21) and (22) and apply the results of (18) to obtain the biases of the T_{ri} and T_{qi} as

$$Bias(T_{ri})=\bar{Y}\gamma\left[\begin{array}{l}\frac{(3\theta_i^2+1)C_\phi^2}{8}-\frac{\theta_i\rho_{y\phi}C_yC_\phi}{2}+a_i\frac{P}{\bar{Y}}\frac{\theta_iC_\phi^2}{2}\\ +b_i\left(1+\frac{3\theta_i^2C_\phi^2}{8}-\frac{\theta_i\rho_{y\phi}C_yC_\phi}{2}\right)\end{array}\right] \quad ...(23)$$

$$Bias(T_{qi})=\bar{Y}\gamma\left[\begin{array}{l}\frac{(3g^2\theta_i^2+1)C_\phi^2}{2}+\frac{g\theta_i\rho_{y\phi}C_yC_\phi}{2}-u_i\frac{P}{\bar{Y}}\frac{g\theta_iC_\phi^2}{2}\\ +v_i\left(1+\frac{3g^2\theta_i^2C_\phi^2}{8}+\frac{g\theta_i\rho_{y\phi}C_yC_\phi}{2}\right)\end{array}\right] \quad ...(24)$$

Squaring and taking expectation of (21) and (22) and apply the results of (18) to obtain the MSE of proposed estimators as T_{ri} and T_{qi} as

$$MSE(T_{ri})=\bar{Y}^2\left(A_i+a_i^2B_i+b_i^2C-2a_iD_i+2b_iE_i-2a_ib_iF_i\right) \quad ...(25)$$

$$MSE(T_{qi})=\bar{Y}^2\left(A_{1i}+u_i^2B_{1i}+v_i^2C_{1i}-2u_iD_{1i}+2v_iE_{1i}-2u_iv_iF_{1i}\right) \quad ...(26)$$

Where $A_i=\gamma\left(C_y^2+\frac{\theta_i^2C_\phi^2}{4}-\theta_i\rho_{y\phi}C_yC_\phi\right)$, $B_i=\gamma\frac{P^2}{\bar{Y}^2}C_\phi^2$,

$C_i=1+\gamma\left(C_y^2+\theta_i^2C_\phi^2-2\theta_i\rho_{y\phi}C_yC_\phi\right)$,

$D_i=\gamma\frac{P}{\bar{Y}}\left(\rho_{y\phi}C_yC_\phi-\frac{\theta_iC_\phi^2}{2}\right)$, $E_i=\gamma\left(C_y^2-\frac{3\theta_i\rho_{y\phi}C_yC_\phi}{2}+\frac{(5\theta_i^2+1)C_\phi^2}{8}\right)$, $F_i=\gamma\frac{P}{\bar{Y}}\left(\rho_{y\phi}C_yC_\phi-\theta_iC_\phi^2\right)$

$D_{1i}=\gamma\frac{P}{\bar{Y}}\left(\rho_{y\phi}C_yC_\phi+\frac{g\theta_iC_\phi^2}{2}\right)$, $E_{1i}=\gamma\left(C_y^2+\frac{3g\theta_i\rho_{y\phi}C_yC_\phi}{2}+\frac{(5g^2\theta_i^2+1)C_\phi^2}{8}\right)$,

$F_{1i}=\gamma\frac{P}{\bar{Y}}\left(\rho_{y\phi}C_yC_\phi+g\theta_iC_\phi^2\right)$.

Differentiating (25) with respect to a_i and b_i, equate to zero and solve for a_i and b_i, simultaneously,

we obtain $a_i=\frac{E_iF_i-C_iD_i}{F_i^2-B_iC_i}$ and . $b_i=\frac{B_iE_i-D_iF_i}{F_i^2-B_iC_i}$

Substituting the results in (25), we obtained the minimum MSE of T_{ri} as

$$MSE(T_{ri})_{\min}=\bar{Y}^2\left(A_i+\left(\frac{C_iD_i^2+B_iE_i^2-2D_iE_iF_i}{F_i^2-B_iC_i}\right)\right) \quad ...(27)$$

Differentiating (26) with respect to u_i and v_i, equate to zero and solve for u_i and v_i simultaneously,

we obtain $u_i=\frac{E_{1i}F_{1i}-C_{1i}D_{1i}}{F_{1i}^2-B_{1i}C_{1i}}$ and $v_i=\frac{B_{1i}E_{1i}-D_{1i}F_{1i}}{F_{1i}^2-B_{1i}C_{1i}}$.

Substituting the results in (26), we obtained the minimum MSE of T_{qi} as

$$MSE(T_{qi})_{\min}=\bar{Y}^2\left(A_{1i}+\left(\frac{C_{1i}D_{1i}^2+B_{1i}E_{1i}^2-2D_{1i}E_{1i}F_{1i}}{F_{1i}^2-B_{1i}C_{1i}}\right)\right) \quad ...(28)$$

Efficiency Comparisons

The suggested estimators T_{ri} and T_{qi} are more efficient than $\bar{y}_m$, $\bar{y}_{s1}$, $\bar{y}_{s2}$, $\bar{y}_{zk}$, $\bar{y}_z$, t_{pi} and t_{qi}, if the following condition are satisfied

$MSE(T_{ri})_{\min}<Var(\bar{y}_m)$ and if $MSE(T_{qi})_{\min}<Var(\bar{y}_m)$

$$A_i+\frac{C_iD_i^2+B_iE_i^2-2D_iE_iF_i}{F_i^2-B_iC_i}<\gamma C_y^2 \quad ...(29)$$

$$A_{1i}+\frac{C_{1i}D_{1i}^2+B_{1i}E_{1i}^2-2D_{1i}E_{1i}F_{1i}}{F_{1i}^2-B_{1i}C_{1i}}<\gamma C_y^2 \quad ...(30)$$

$MSE(T_{ri})_{\min}<MSE(\bar{y}_{s1})$ and $MSE(T_{qi})_{\min}<MSE(\bar{y}_{s1})$ if

$$A_i+\frac{C_iD_i^2+B_iE_i^2-2D_iE_iF_i}{F_i^2-B_iC_i}<\gamma\left(C_y^2+\frac{C_\phi^2}{4}-\rho_{y\phi}C_yC_\phi\right) \quad ...(31)$$

$$A_{1i}+\frac{C_{1i}D_{1i}^2+B_{1i}E_{1i}^2-2D_{1i}E_{1i}F_{1i}}{F_{1i}^2-B_{1i}C_{1i}}<\gamma\left(C_y^2+\frac{C_\phi^2}{4}-\rho_{y\phi}C_yC_\phi\right) \quad ...(32)$$

$MSE(T_{ri})_{\min}<MSE(\bar{y}_{s2})_{\min}$ and $MSE(T_{qi})_{\min}<MSE(\bar{y}_{s2})_{\min}$ if

$$A_i+\frac{C_iD_i^2+B_iE_i^2-2D_iE_iF_i}{F_i^2-B_iC_i}<\gamma\left(C_y^2+\frac{C_\phi^2}{4}-\rho_{y\phi}C_yC_\phi\right) \quad ...(33)$$

$$A_{1i}+\frac{C_{1i}D_{1i}^2+B_{1i}E_{1i}^2-2D_{1i}E_{1i}F_{1i}}{F_{1i}^2-B_{1i}C_{1i}}<\gamma C_y^2\left(1-\rho_{y\phi}^2\right) \quad ...(34)$$

$MSE(T_{ri})_{\min}<MSE(\bar{y}_{zk})$ and $MSE(T_{qi})_{\min}<MSE(\bar{y}_{zk})$ if

$$A_i+\frac{C_iD_i^2+B_iE_i^2-2D_iE_iF_i}{F_i^2-B_iC_i}<\gamma\left(C_y^2+\theta_i^2C_\phi^2-2\theta_i\rho_{y\phi}C_yC_\varphi\right) \quad ...(35)$$

$$A_{1i}+\frac{C_{1i}D_{1i}^2+B_{1i}E_{1i}^2-2D_{1i}E_{1i}F_{1i}}{F_{1i}^2-B_{1i}C_{1i}}<\gamma\left(C_y^2+\theta_i^2C_\phi^2-2\theta_i\rho_{y\phi}C_yC_\varphi\right) \quad ...(36)$$

$MSE(T_{ri})_{\min}<MSE(\bar{y}_{zi})_{\min}$ and $MSE(T_{qi})_{\min}<MSE(\bar{y}_{zi})_{\min}$ if

$$A_i+\frac{C_iD_i^2+B_iE_i^2-2D_iE_iF_i}{F_i^2-B_iC_i}<\gamma C_y^2\left(1-\rho_{y\phi}^2\right) \quad ...(37)$$

$$A_{1i}+\frac{C_{1i}D_{1i}^2+B_{1i}E_{1i}^2-2D_{1i}E_{1i}F_{1i}}{F_{1i}^2-B_{1i}C_{1i}}<\gamma C_y^2\left(1-\rho_{y\phi}^2\right) \quad ...(38)$$

$MSE(T_{ri})_{\min}<MSE(t_{pi})$ and $MSE(T_{qi})_{\min}<MSE(t_{pi})$ if

$$A_i+\frac{C_iD_i^2+B_iE_i^2-2D_iE_iF_i}{F_i^2-B_iC_i}<\gamma\left(C_y^2+\Delta_i^2C_\phi^2-2\Delta_i\rho_{y\phi}C_yC_\varphi\right) \quad ...(39)$$

$$A_{1i}+\frac{C_{1i}D_{1i}^2+B_{1i}E_{1i}^2-2D_{1i}E_{1i}F_{1i}}{F_{1i}^2-B_{1i}C_{1i}}<\gamma\left(C_y^2+\Delta_i^2C_\phi^2-2\Delta_i\rho_{y\phi}C_yC_\varphi\right) \quad ...(40)$$

Where, $\Delta=w_1+\frac{b_\phi P}{\bar{Y}}$.

$MSE(T_{ri})_{\min}<MSE(t_{qi})$ and $MSE(T_{qi})_{\min}<MSE(t_{qi})$ if

$$A_i+\frac{C_iD_i^2+B_iE_i^2-2D_iE_iF_i}{F_i^2-B_iC_i}<\gamma\left(C_y^2+\frac{b_\phi^2P^2C_\phi^2}{\bar{Y}^2}-\frac{2P\rho_{y\phi}b_\phi C_yC_\varphi}{\bar{Y}}\right) \quad ...(41)$$

$$A_{1i}+\frac{C_{1i}D_{1i}^2+B_{1i}E_{1i}^2-2D_{1i}E_{1i}F_{1i}}{F_{1i}^2-B_{1i}C_{1i}}<\gamma\left(C_y^2+\frac{b_\phi^2P^2C_\phi^2}{\bar{Y}^2}-\frac{2P\rho_{y\phi}b_\phi C_yC_\varphi}{\bar{Y}}\right) \quad ...(42)$$

Empirical Study

In this section, the suggested estimators T_{ri} and T_{qi} performance are assessed with that of the sample mean $\bar{y}_m$, Audu *et al.*[1] estimators, t_{pi} and t_{qi} numerically considering two natural populations used as.

Population 1: Zaman[13]

$$\phi_i=\begin{cases}1, & \textit{if a circle consists of more than five villages}\\ 0, & \textit{otherwise}\end{cases}$$

$$N=89, n=20, \bar{Y}=3.3596, P=0.1236, \beta_{2(\phi)}=3.492, C_y=0.6008, C_\phi=2.6779, \rho_{y\phi}=0.766.$$

Population 2: Zaman[13]

$$\phi_i=\begin{cases}1, & \textit{if the numbe of teachers is more than sixty}\\ 0, & \textit{otherwise}\end{cases}$$

$$N=111, n=30, \bar{Y}=29.279, P=0.117, \beta_{2(\phi)}=3.898, C_y=0.872, C_\phi=2.758, \rho_{y\phi}=0.797,$$

Table 3: MSEs and PREs of suggested estimators and existing ones using population 1

Estimators	MSE	PRE	Estimators	MSE	PRE
Sample mean estimator			$\bar{y}_m$	0.1579298	100.00
		Audu *et al.*[1] Estimators			
t_{p1}	0.0661802	238.636	t_{p2}	0.06679036	236.456
t_{p3}	0.08040544	196.4168	t_{p4}	0.08037597	196.4888
t_{p5}	0.07114321	221.9886	t_{p6}	0.1366728	115.5532
t_{p7}	0.0661782	238.6432	t_{p8}	0.167143	94.48783
t_{p9}	0.06581011	239.978	t_{qi}	0.06526354	241.9878

		Suggested Estimators			
T_{r1}	0.001670981	9451.3223	T_{q1}	0.044078	358.2962
T_{r2}	0.0467923	337.5124	T_{q2}	0.04682617	337.2682
T_{r3}	0.04676769	337.6900	T_{q3}	0.0468241	337.2832
T_{r4}	0.04621405	341.7355	T_{q4}	0.04677797	337.6158
T_{r5}	0.04621526	341.7265	T_{q5}	0.04677807	337.6150
T_{r6}	0.04659158	338.9664	T_{q6}	0.04680936	337.3894
T_{r7}	0.04383939	360.2463	T_{q7}	0.04658672	339.0018
T_{r8}	0.0467924	337.5116	T_{q8}	0.04682618	337.2682
T_{r9}	0.04245525	371.6411	T_{q9}	0.04648274	339.7601
T_{r10}	0.04680724	337.4046	T_{q10}	0.04682742	337.2592

Table 4: MSEs and PREs of suggested estimators and existing ones using population 2

Estimators	MSE	PRE	Estimators	MSE	PRE
Sample mean estimator		$\bar{y}_m$		15.85573	100.00
		Audu *et al.*[1] Estimators			
t_{p1}	5.817701	272.5429	t_{p2}	5.849699	271.0521
t_{p3}	6.4338	246.4443	t_{p4}	6.582441	240.8792
t_{p5}	6.015808	263.5678	t_{p6}	9.077466	174.6713
t_{p7}	5.826441	272.1341	t_{p8}	11.03681	143.6623
t_{p9}	5.805672	273.1076	t_{qi}	5.784028	274.1296
		Suggested Estimators			
T_{r1}	3.39183	467.4683	T_{q1}	4.854043	326.6500
T_{r2}	5.023029	315.6607	T_{q2}	5.023891	315.6066
T_{r3}	5.022078	315.7205	T_{q3}	5.023761	315.6147
T_{r4}	5.004624	316.8216	T_{q4}	5.021381	315.7643
T_{r5}	5.000151	317.1050	T_{q5}	5.020775	315.8024
T_{r6}	5.017135	316.0316	T_{q6}	5.023085	315.6572
T_{r7}	4.9231	322.0645	T_{q7}	5.010563	316.4461
T_{r8}	5.022769	315.6771	T_{q8}	5.023856	315.6088
T_{r9}	4.86016	326.2388	T_{q9}	5.002496	316.9564
T_{r10}	5.023386	315.6383	T_{q10}	5.02394	315.6035

Table 3 and 4 show the Mean Square Errors and Percentage Relative Efficiencies of the sample mean, $\bar{y}_m$, Audu *et al.,*[1] t_{pi} and T_{qi}, and suggested estimators, T_{ri} and T_{qi} estimators, considering two data sets respectively. The results revealed that the suggested estimators T_{ri} and T_{qi} have minimum MSEs and higher PREs as compared to the sample mean, Audu *et al.*[1] estimators.

Results and Discussion

An improved classes of regression type estimators of finite population mean are suggested. Table 3 shows MSEs and PREs of the suggested and some existing estimators using dataset 1. The result shows that the suggested estimators have minimum MSEs and higher PREs compared to the conventional estimators and Audu *et al.*[1] estimators.

Table 4 shows MSEs and PREs of the suggested and some existing estimators using dataset 2. The result shows that the suggested estimators have minimum MSEs and higher PREs compared to the conventional estimators and Audu *et al.*[1] estimators.

Conclusion

In this research, we proposed an improved modified regression estimators for the estimation of population mean in the presence of auxiliary attribute. The results of the empirical study revealed that the proposed estimators are more efficient than sample mean and Audu *et al.*[1] estimators. This implies that the proposed estimators have great chance of producing precise estimate.

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Conflict of Interest

The authors declare no conflict of interest.

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