ISSN: 2456-799X, Vol.07, No.(2) 2022, Pg. 67-73

Oriental Journal of Physical Sciences

www.orientjphysicalsciences.org

On the Arithmetic Mean Estimators of a Family of Estimators of Finite Population Variance of Ratio Type

MOJEED ABIODUN YUNUSA[1], AHMED AUDU[1] and JAMIU OLASUNKANMI MUILI[2]*

[1]Department of Statistics, Usmanu Danfodiyo University, Sokoto, Nigeria.
[2]Department of Mathematics, Kebbi State University of Science and Technology Aliero, Nigeria.

Abstract
For many years, one of the difficult components of sampling theory has been the estimation of population characteristics, especially variance. The estimation of variability is very essential in many fields (Chemistry, Biology, Mathematics, and so on) to know how one quantity varies with respect to another quantity. This paper proposes arithmetic estimators of a group of ratio estimators for populations with finite variance.Using a Taylor series technique, the bias and MSE of the proposed estimators are determined up to the first order of approximation together with the efficiency conditions overexisting estimators. The effectiveness of the proposed estimators in comparison to the current estimators is evaluated using a real-world data set. The empirical findings demonstrate that the suggested estimators outperform the current estimators taken into account in the study. Hence, these suggested estimators are recommended for use in real life scenario.

Article History
Received: 03 August 2022
Accepted: 20 October 2022

Keywords:
Arithmetic Mean;
Bias; Efficiency;
Mean Square Error;
Ratio estimator.

Introduction

Isaki[1] was the first to discuss the issue of estimates of population variance when auxiliary variable is present and proposed the classical ratio method of estimation for population variance which utilize the ratio population and sample variance of auxiliary variable as auxiliary information which is strongly correlated with the study variable. Numerous authors have modified ratio estimators for finite population variance when an auxiliary variable is present in the literature, authors like Evans,[3] Hansen and Hurwitz,[5] Prasad and Singh.[14] Prasad and Singh[14] modified thework of Isaki[6] by incorporating coefficient of kurtosis into his work. Ever since then several authors have been making efforts to develop estimators with greater efficiency using auxiliary information for improvement and modification, other authors include Kadilar and Cingi,[7] Gupta and Shabbir,[4] Tailor and Sharma,[16] Khan and Shabbir,[8] Yadav *et al.,*[17] Subramani,[15] Audu *et al.,*[1] Maqbool and Shakeel,[9] Muili *et al.,*[12,11] Bhat *et al.,*[2] and Muili *et al.*[10] Muili *et al.*[10] modified the work of Bhat

CONTACT Jamiu Olasunkanmi Muili jamiunice@yahoo.com Department of Mathematics, Kebbi State University of Science and Technology Aliero, Nigeria.

Doi: http://dx.doi.org/10.13005/OJPS07.02.03

et al.[2] by incorporating First quartile, Downton's method, Deciles, Mid-range and Percentiles into the developed estimators for the purposed of precision.

To provide estimates that are closer to the population mean of the study variable, we proposed arithmetic mean estimators from a family of ratio type estimators of finite population variance in this study.

Consider a finite population G={G_1, G_2..., G_N}, a set of N units, where G_i={X_i, Y_i}, i = 1,2,...,N has a pair values. Y is the study variable and is associated with the auxiliary variable X, where Y={Y_1, Y_2..., Y_N} and x={x_1, x_2..., x_N} are the n sample values. The sample means for the study and the auxiliary variables are $\bar{y}$ and $\bar{x}$, respectively. S_x^2 and S_y^2 are the population mean squares of X and Y respectively and s_x^2 and s_y^2 represent the sample mean square for the size n randomly selected, without replacement. Y: Study variable,n: sample size, N: population size, X: Auxiliary variable, $\bar{x}$, $\bar{y}$: Auxiliary and study variables sample means, $\bar{x}$, $\bar{y}$: Auxiliary and study variables population means, f: Sampling fraction, p : Correlation coefficient, C_y, C_x : Coefficient of variation of study and auxiliary variable, $\beta_{1(x)}$: Skewness, Q_1: First quartile of auxiliary variable, Q_2 : Second quartile, Q_3 :Third quartile, QD : Quartile deviation, $\beta_{2(x)}$: Kurtosis, TM : Tri-mean, M_d : Median of auxiliary variable, M_x : Maximum value of auxiliary variable, Q_1: Hodges lehman estimator, MR : Population mid-range of auxiliary variable, Q_1: Downton's method, P_1: Percentile of auxiliary variable.

$$\bar{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i,\ \bar{X}=\frac{1}{N}\sum_{i=1}^{N}X_i,\ \bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i,\ \bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i,\ \gamma=\frac{1-f}{n},\ f=\frac{n}{N},$$

$$MR=\frac{X_{(1)}+X_{(2)}}{2},$$

$$TM=\frac{(Q_1+2Q_2+Q_3)}{4},\ S_y^2=\frac{1}{N}\sum_{i=1}^{N}(Y_i-\bar{Y})^2,\ S_x^2=\frac{1}{N}\sum_{i=1}^{N}(X_i-\bar{X})^2,$$

$$s_x^2=\frac{1}{n-1}\sum_{i=1}^{n}(y_i-y)^2,\ s_x^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2,$$

$$HL=Median\left(\frac{X_i+X_j}{2},\ i\le i\le j\le N\right),$$

$$D=\frac{2\sqrt{\pi}}{N(N-1)}\sum_{i=1}^{N}\left(i-\frac{N+1}{2}\right)X_i$$

Literature Review

The population's variance's unbiased estimator is

$$t_0=S_y^2 \quad ...(1)$$

The estimator's bias and mean square error (Variance) are provided by

$$Bias(t_0)=0 \quad ...(2)$$

$$Var(t_0)=\gamma S_y^4\left(\beta_{2(y)}-1\right) \quad ...(3)$$

When the population and sample variance of the auxiliary variable are available, Isaki[6]'s proposed ratio estimator for estimating population variance of the study variable is defined as

$$t_R=s_y^2\left(\frac{S_x^2}{s_x^2}\right) \quad ...(4)$$

The estimator's t_R bias and mean square error are

$$Bias(t_R)=\gamma S_y^2\left(\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right) \quad ...(5)$$

$$MSE(t_R)=\gamma S_y^4\left(\left(\beta_{2(y)}-1\right)+\left(\beta_{2(x)}-1\right)-2\left(\lambda_{22}-1\right)\right) \quad ...(6)$$

Kadilar and Cingi[7] developed class of ratio estimators taking into account the coefficient of variation and kurtosis of the auxiliary variable as auxiliary information for improvement in the estimation of population variance and it is defined as

$$t_{KCi}=s_y^2\left(\frac{S_x^2+l_i}{s_x^2+l_i}\right),\quad i=1,2,3,4 \quad ...(7)$$

The estimator's t_{KCi} bias and mean square error are,

$$Bias(t_{KCi})=\gamma L_i S_y^2\left(L_i\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right),\quad i=1,2,3,4 \quad ...(8)$$

$$MSE(t_{KCi})=\gamma S_y^4\left(\left(\beta_{2(y)}-1\right)+L_i^2\left(\beta_{2(x)}-1\right)-2L_i\left(\lambda_{22}-1\right)\right),\quad i=1,2,3,4 \quad ...(9)$$

where, $l_1=C_x$, $l_2=\beta_{2(x)}$, $l_4=\beta_{2(x)}/C_x$, $l_3=C_x/\beta_{2(x)}$,

$$L_i=\frac{S_x^2}{S_x^2+l_i},\ i=1,2,3,4.$$

Subramani[15] developed a generalized modified ratio estimator that uses the median and coefficient of variation of the auxiliary variable to estimate the finite population variance of the study variable in order to enhance the precision of the estimate as.

$$t_S = s_y^2\left(\frac{S_x^2+k}{s_x^2+k}\right) \quad ...(10)$$

The estimator's t_S bias and mean square error are,

$$Bias(t_S) = \gamma K S_y^2\left(K\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right) \quad ...(11)$$

$$MSE(t_S) = \gamma S_y^4\left(\left(\beta_{2(y)}-1\right)+K^2\left(\beta_{2(x)}-1\right)-2K\left(\lambda_{22}-1\right)\right) \quad ...(12)$$

where, $k = M_d / C_x$, $K=\frac{S_x^2}{S_x^2+k}$.

Bhat *et al.*[2] modified robust estimators for the estimation of population variance utilizing a linear combination of Downton's approach and deciles of the auxiliary variable in order to improve the precision of the estimate as well as the estimation of the study's finite population variance.

$$t_{Bi} = s_y^2\left(\frac{S_x^2+b_i}{s_x^2+b_i}\right), \quad i=1,2,.....,10 \quad ...(13)$$

The estimator's t_{Bi} bias and mean square error are

$$Bias(t_{Bi}) = \gamma B_i S_y^2\left(B_i\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right), \quad i=1,2,...,10 \quad ...(14)$$

$$MSE(t_{KCi}) = \gamma S_y^4\left(\left(\beta_{2(y)}-1\right)+\left(\beta_{2(x)}-1\right)-2\left(\lambda_{22}-1\right)\right), \quad i=1,2,3,4 \quad ...(15)$$

where, $b_1 = (D+D_1)$, $b_2 = (D+D_2)$, $b_3 = (D+D_3)$, $b_4 = (D+D_4)$ $b_5 = (D+D_5)$, $b_6 = (D+D_6)$, $b_7 = (D+D_7)$, $b_8 = (D+D_8)$, $b_9 = (D+D_9)$, $b_{10} = (D+D_{10})$,

$$B_i = \frac{S_x^2}{S_x^2+b_i}, \ i=1,2,...,10$$

Muili *et al.*[10] developed a family of ratio type estimators for finite population variance estimation utilizing the parameters of the auxiliary variable, first quartile, downton's method, deciles, mid-range and percentiles as

$$t_{Ji} = s_y^2\left(\frac{S_x^2+h_i}{s_x^2+h_i}\right), \quad i=1,2,...,21 \quad ...(16)$$

The estimator's t_{ji} bias and mean square error are:

$$Bias(t_{Ji}) = \gamma H_i S_y^2\left(H_i\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right), \quad i=1,2,...,21 \quad ...(17)$$

$$MSE(t_{Ji}) = \gamma S_y^4\left(\left(\beta_{2(y)}-1\right)+H_i^2\left(\beta_{2(x)}-1\right)-2H_i\left(\lambda_{22}-1\right)\right), \quad i=1,2,...,21 \quad ...(18)$$

Where, $h_1 = (D_{10}+P_1)$, $h_2 = (D_{10}+P_5)$, $h_3 = (D_{10}+P_{10})$, $h_4 = (D_{10}+P_{15})$, $h_5 = (D_{10}+P_{20})$, $h_6 = (D_{10}+P_{25})$, $h_7 = (D_{10}+P_{30})$, $h_8 = (D_{10}+P_{35})$, $h_9 = (D_9+P_{40})$, $h_{10} = (MR+P_{45})$, $h_{11} = (MR+P_{50})$, $h_{12} = (MR+P_{55})$, $h_{13} = (MR+P_{60})$, $h_{14} = (MR+P_{65})$, $h_{15} = (\bar{X}+P_{70})$, $h_{16} = (\bar{X}+P_{75})$, $h_{17} = (D+P_{18})$, $h_{18} = (D+P_{85})$, $h_{19} = (D+P_{19})$, $h_{20} = (D+P_{95})$, $h_{21} = (Q+P_{99})$

$$H_i = \frac{S_x^2}{S_x^2+h_i}, \ i=1,2,...,21.$$

Proposed Estimators

We proposed arithmetic mean estimators of family of ratio type estimators of finite population variance when correlation between study and auxiliary variable is either positive or negative after studying and being inspired by Muili *et al.*[10]'s work.

$$t_{A1} = 2^{-1}(1+z_1)s_y^2\left(\frac{S_x^2+(D_{10}+P_1)}{s_x^2+(D_{10}+P_1)}\right) \quad ...(19)$$

$$t_{A2} = 2^{-1}(1+z_2)s_y^2\left(\frac{S_x^2+(D_{10}+P_5)}{s_x^2+(D_{10}+P_5)}\right) \quad ...(20)$$

$$t_{A3} = 2^{-1}(1+z_3)s_y^2\left(\frac{S_x^2+(D_{10}+P_{10})}{s_x^2+(D_{10}+P_{10})}\right) \quad ...(21)$$

$$t_{A4} = 2^{-1}(1+z_4)s_y^2\left(\frac{S_x^2+(D_{10}+P_{15})}{s_x^2+(D_{10}+P_{15})}\right) \quad ...(22)$$

$$t_{A5} = 2^{-1}(1+z_5)s_y^2\left(\frac{S_x^2+(D_{10}+P_{20})}{s_x^2+(D_{10}+P_{20})}\right) \quad ...(23)$$

$$t_{A6} = 2^{-1}(1+z_6)s_y^2\left(\frac{S_x^2+(D_{10}+P_{25})}{s_x^2+(D_{10}+P_{25})}\right) \quad ...(24)$$

$$t_{A7} = 2^{-1}(1+z_7)s_y^2\left(\frac{S_x^2+(D_{10}+P_{30})}{s_x^2+(D_{10}+P_{30})}\right) \quad ...(25)$$

$$t_{A8} = 2^{-1}(1+z_8)s_y^2\left(\frac{S_x^2+(D_{10}+P_{35})}{s_x^2+(D_{10}+P_{35})}\right) \quad ...(26)$$

$$t_{A9} = 2^{-1}(1+z_9)s_y^2\left(\frac{S_x^2+(D_9+P_{40})}{s_x^2+(D_9+P_{40})}\right) \quad ...(27)$$

$$t_{A10} = 2^{-1}(1+z_{10})s_y^2\left(\frac{S_x^2+(MR+P_{45})}{s_x^2+(MR+P_{45})}\right) \quad ...(28)$$

$$t_{A11} = 2^{-1}(1+z_{11})s_y^2\left(\frac{S_x^2+(MR+P_{50})}{s_x^2+(MR+P_{50})}\right) \quad ...(29)$$

$$t_{A12} = 2^{-1}(1+z_{12})s_y^2\left(\frac{S_x^2+(MR+P_{55})}{s_x^2+(MR+P_{55})}\right) \quad ...(30)$$

$$t_{A13} = 2^{-1}(1+z_{13})s_y^2\left(\frac{S_x^2+(MR+P_{60})}{s_x^2+(MR+P_{60})}\right) \quad ...(31)$$

$$t_{A14} = 2^{-1}(1+z_{14})s_y^2\left(\frac{S_x^2+(MR+P_{65})}{s_x^2+(MR+P_{65})}\right) \quad ...(32)$$

$$t_{A15} = 2^{-1}(1+z_{15})s_y^2\left(\frac{S_x^2+(\bar{X}+P_{70})}{s_x^2+(\bar{X}+P_{70})}\right) \quad ...(33)$$

$$t_{A16} = 2^{-1}(1+z_{16})s_y^2\left(\frac{S_x^2+(\bar{X}+P_{75})}{s_x^2+(\bar{X}+P_{75})}\right) \quad ...(34)$$

$$t_{A17} = 2^{-1}(1+z_{17})s_y^2\left(\frac{S_x^2+(D+P_{80})}{s_x^2+(D+P_{80})}\right) \quad ...(35)$$

$$t_{A18} = 2^{-1}(1+z_{18})s_y^2\left(\frac{S_x^2+(D+P_{85})}{s_x^2+(D+P_{85})}\right) \quad ...(36)$$

$$t_{A19} = 2^{-1}(1+z_{19})s_y^2\left(\frac{S_x^2+(D+P_{90})}{s_x^2+(D+P_{90})}\right) \quad ...(37)$$

$$t_{A20} = 2^{-1}(1+z_{20})s_y^2\left(\frac{S_x^2+(D+P_{95})}{s_x^2+(D+P_{95})}\right) \quad ...(38)$$

$$t_{A21} = 2^{-1}(1+z_{21})s_y^2\left(\frac{S_x^2+(Q_1+P_{99})}{s_x^2+(Q_1+P_{99})}\right) \quad ...(39)$$

The suggested estimators can be expressed generally as follows,

$$t_{Ai} = 2^{-1}(1+z_i)s_y^2\left(\frac{S_x^2+h_i}{s_x^2+h_i}\right), \; i=1,2,...,21 \quad ...(40)$$

Properties (Bias and MSE) of the Proposed Estimators

To derive the bias and MSE of the proposed estimators we use the following definitions e's sampling errors as

$e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ such that $s_x^2 = S_x^2(1+e_1)$

and $s_x^2 = S_x^2(1+e_1)$,

$E(e_0)=E(e_1)=0$, $E(e_0^2)=\gamma(\beta_{2(y)}-1)$, $E(e_1^2)=\gamma(\beta_{2(x)}-1)$ and

$E(e_0e_1)=\gamma(\lambda_{22}-1)$

Expressing (40) in terms of sampling errors, we have

$$t_{Ai} = 2^{-1}(1+z_i)S_y^2(1+e_0)\left(\frac{S_x^2+h_i}{S_x^2(1+e_1)+h_i}\right) \quad ...(41)$$

$$t_{Ai} = 2^{-1}(1+z_i)S_y^2(1+e_0)\left(\frac{S_x^2+h_i}{S_x^2+h_i+S_x^2e_1}\right) \quad ...(42)$$

$$t_{Ai} = 2^{-1}(1+z_i)S_y^2(1+e_0)(1+\Delta_ie_1)^{-1} \quad ...(43)$$

Where, $\Delta_i = \frac{S_x^2}{S_x^2+h_i}$, $i=1,2,....,21$

By simplifying (43) up to first order approximation, we have

$$t_{Ai} = 2^{-1}(1+z_i)S_y^2(1+e_0)(1-\Delta_ie_1+\Delta_i^2e_1^2) \quad ...(44)$$

By expanding, simplifying and subtracting S^2_y from both sides of (44), we have

$$t_{Ai} - S_y^2 = S_y^2\left((1+z_i)\left(\frac{1+e_0-\Delta_ie_1+\Delta_i^2e_1^2-\Delta_ie_0e_1}{2}\right)-1\right) \quad ...(45)$$

By taking the expectations from both sides of (45), one may get the estimator's t_{Ai} bias as

$$E(t_{Ai}-S_y^2) = S_y^2E\left((1+z_i)\left(\frac{1+e_0-\Delta_ie_1+\Delta_i^2e_1^2-\Delta_ie_0e_1}{2}\right)-1\right) \quad ...(46)$$

$$Bias(t_{Ai}) = S_y^2\left(2^{-1}(1+z_i)\left[1+\gamma\left(\Delta_i^2(\beta_{2(x)}-1)-\Delta_i(\lambda_{22}-1)\right)\right]-1\right) \quad ...(47)$$

The MSE of t_{Ai} is obtained by squaring both sides of (45) and taking expectation.

$$MSE(t_{Ai}) = S_y^4\left(\begin{array}{l}1+(1+z_i)^2\left(\frac{1+\gamma\left[(\beta_{2(y)}-1)+3\Delta_i^2(\beta_{2(x)}-1)-4\Delta_i(\lambda_{22}-1)\right]}{4}\right)\\ -(1+z_i)\left[1+\gamma\left(\Delta_i^2(\beta_{2(x)}-1)-\Delta_i(\lambda_{22}-1)\right)\right]\end{array}\right) \quad ...(48)$$

By solving for z_i and differentiating (48) with respect to equatingto zero, we obtain

$$z_i == \frac{2A_i}{B_i}-1, \quad i=1,2,...,21 \quad ...(49)$$

where, $A_i = 1+\gamma\left[\Delta_i^2(\beta_{2(x)}-1)-\Delta_i(\lambda_{22}-1)\right]$ and

$B_i = 1+\gamma\left[(\beta_{2(y)}-1)+3\Delta_i^2(\beta_{2(x)}-1)-4\Delta_i(\lambda_{22}-1)\right]$

By substituting (49) into (48), getting the estimator's minimum MSE t_{AI} as

$$MSE(t_{Ai})_{\min} = S_y^2\left(1-\frac{A_i^2}{B_i}\right), \quad i=1,2,..,21 \quad ...(50)$$

Efficiency Comparisons

The proposed estimators t_{AI} are more efficient than the existing estimators, if the following conditions are satisfied

$MSE(t_{Ai})_{\min} < Var(t_0)$, if

$$\gamma^{-1}\left(1-\frac{A_i^2}{B_i}\right)<\left(\beta_{2(y)}-1\right) \quad ...(51)$$

$MSE\left(t_{Ai}\right)_{\min} < MSE\left(t_R\right)$, if

$$\gamma^{-1}\left(1-\frac{A_i^2}{B_i}\right)-\left(\beta_{2(y)}-1\right)<\left(\left(\beta_{2(x)}-1\right)-2\left(\lambda_{22}-1\right)\right) \quad ...(52)$$

$MSE\left(t_{Ai}\right)_{\min} < MSE\left(t_{KCi}\right)$, if

$$\gamma^{-1}\left(1-\frac{A_i^2}{B_i}\right)-\left(\beta_{2(y)}-1\right)<L_i\left(L_i\left(\beta_{2(x)}-1\right)-2\left(\lambda_{22}-1\right)\right) \quad ...(53)$$

$MSE\left(t_{Ai}\right)_{\min} < MSE\left(t_S\right)$, if

$$\gamma^{-1}\left(1-\frac{A_i^2}{B_i}\right)-\left(\beta_{2(y)}-1\right)<K\left(K\left(\beta_{2(x)}-1\right)-2\left(\lambda_{22}-1\right)\right) \quad ...(54)$$

$MSE\left(t_{Ai}\right)_{\min} < MSE\left(t_{Bi}\right)$, if

$$\gamma^{-1}\left(1-\frac{A_i^2}{B_i}\right)-\left(\beta_{2(y)}-1\right)<B_i\left(B_i\left(\beta_{2(x)}-1\right)-2\left(\lambda_{22}-1\right)\right) \quad ...(55)$$

$MSE\left(t_{Ai}\right)_{\min} < MSE\left(t_{Ji}\right)$, if

$$\gamma^{-1}\left(1-\frac{A_i^2}{B_i}\right)-\left(\beta_{2(y)}-1\right)<H_i\left(H_i\left(\beta_{2(x)}-1\right)-2\left(\lambda_{22}-1\right)\right) \quad ...(56)$$

Empirical Study

To evaluate the effectiveness of the proposed estimators over existing ones, empirical study is carried out using a real life data set as

Data

[Source: {Murthy,[13] P.228]

X: Fixed capital

Y: Output of Eighty factories

$N=80$, $n=20$, $S_y= 18.3569$, $S_x=8.4542$, $C_y=0.3542$, $C_x=0.7507$, $\bar{Y}= 51.8264$, $\bar{X}=11.2624$, $\beta_{2(y)}=2.2667$, $\beta_{2(x)}=2.8664$, $\beta_{1(x)}=1.05$, $p=0.9413$, $\lambda_{22}=2.2209$, $Q_1=9.318$, $Q_2=7.575$, $Q_3=16.975$, $D=8.0138$, $MR=17.955$, $D_1=3.6$, $D_2=4.6$, $D_3=5.9$, $D_4=6.7$, $D_5=7.5$, $D_6=8.5$, $D_7=14.8$, $D_8=18.1$, $D_9=25$, $D_{10}=34.8$, $P_1=2.44$, $P_5=4.35$, $P_{10}=5.9$, $P_{15}=6.63$, $P_{20}=7.45$, $P_{25}=7.8$, $P_{30}=8.7$, $P_{35}=11.6$, $P_{40}=15.3$, $P_{50}=17.2$, $P_{55}=19.3$, $P_{60}=21.7$, $P_{65}=23.55$, $P_{70}=24.98$, $P_{75}=25$, $P_{80}=26.95$, $P_{85}=27.8$, $P_{90}=29.7$, $P_{95}=30$, $P_{99}=34.85$, $\Delta_1=0.6574$, $\Delta_2=0.6461$, $\Delta_3=0.6372$, $\Delta_4=0.6330$, $\Delta_5=0.6285$, $\Delta_6=0.6266$, $\Delta_7=0.6217$, $\Delta_8=0.6064$, $\Delta_9=0.6394$, $\Delta_{10}=0.6722$, $\Delta_{11}=0.67030$, $\Delta_{12}=0.66574$, $\Delta_{13}=0.6432$, $\Delta_{14}=0.6326$, $\Delta_{15}=0.66635$, $\Delta_{16}=0.6634$, $\Delta_{17}=0.6715$, $\Delta_{18}=0.6662$, $\Delta_{19}=0.66546$, $\Delta_{20}=0.6528$, $\Delta_{21}=0.6181$,

Table 1: MSEs and PREs of Existing and Proposed Estimators

Estimators	MSE	PRE	Estimators	MSE	PRE
t_0	53893.89	100	t_R	2943.71	183.2344
Kadilar and Cingi[7] Estimator 1			**Subramani[15] Estimator**		
t_{KC1}	2887.46	186.804	t_S	2389.24	225.7576
Bhat *et al.*[2] Estimators					
t_{B1}	2330.10	231.4875	t_{B6}	2191.71	246.1042
t_{B2}	2297.74	234.7476	t_{B7}	2078.86	259.4638
t_{B3}	2258.56	238.8199	t_{B8}	2041.82	264.1707
t_{B4}	2236.84	241.1388	t_{B9}	1999.23	269.7984
t_{B5}	2215.55	243.456	t_{B10}	1999.24	269.797
Muili *et al.*[10] Estimators			**Proposed Estimators**		
t_{J1}	1993.16	270.62	t_{A1}	1958.19	275.4528
t_{J2}	1993.57	270.5644	t_{A2}	1960.58	275.1171
t_{J3}	1995.33	270.3257	t_{A3}	1963.75	274.6729

t_{J4}	1998.26	269.9293	t_{A4}	1965.65	274.4074
t_{J5}	1994.76	270.403	t_{A5}	1967.96	274.0853
t_{J6}	1995.69	270.2769	t_{A6}	1969.03	273.9364
t_{J7}	1995.17	270.3474	t_{A7}	1972.02	273.5211
t_{J8}	1993.16	270.62	t_{A8}	1983.63	271.9202
t_{J9}	1994.01	270.5047	t_{A9}	1962.86	274.7975
t_{J10}	1996.71	270.1389	t_{A10}	1957.84	275.5021
t_{J11}	1993.79	270.5245	t_{A11}	1957.71	275.5204
t_{J12}	1993.77	270.5372	t_{A12}	1957.18	275.5950
t_{J13}	1995.50	270.3027	t_{A13}	1961.48	274.9908
t_{J14}	1994.24	270.4735	t_{A14}	1965.84	274.3809
t_{J15}	1993.07	270.6322	t_{A15}	1957.67	275.5260
t_{J16}	1993.08	270.6309	t_{A16}	1957.68	275.5246
t_{J17}	1994.39	270.4531	t_{A17}	1957.79	275.5091
t_{J18}	1994.16	270.62	t_{A18}	1958.16	275.4571
t_{J19}	1993.57	270.5644	t_{A19}	1960.05	275.1914
t_{J20}	1995.33	270.3257	t_{A20}	1958.94	275.3474
t_{J21}	1998.26	269.9293	t_{A21}	1974.44	273.1858

Results and Discussion

Using the aforementioned real-life dataset, Table 1 displays the MSEs and PREs of the proposed and existing estimators. The findings showed that, in comparison to the existing estimators, the proposed ones have lower MSEs and greater PREs. This suggests that the proposed estimators have a better chance of generating estimates that are more in line with the study variable's actual population mean.

Conclusion

In this study, we proposed arithmetic mean estimators of a family of ratio type estimators of finite population variance, the properties (Biases and MSEs) of the proposed estimators were derived and from the results of empirical study, it was obtained that the proposed estimators are more efficient than sample mean estimator, ratio estimator, Kadilar and Cingi[7] estimator, Subramani[15] estimator, Bhat *et al.*[2] and Muili *et al.*[10] estimators. Hence, the proposed estimators are recommended for use in real life situation.

Acknowledgement

The author would like to thank, Department of Mathematics, Kebbi State University of Science and Technology Aliero, Nigeria. for their guidance and support to complete this article.

Funding

No financial supported was received by the authors for the research, authorship and publication of this article.

Conflict of Interest

The authors declare no conflict of interest.

References

1. Audu A., Adewara A. A., and R. V. K., Class of Ratio Estimator with Known Functions of Auxiliary Variables for Estimating Finite Population Variance. *Asian Journal of Mathematics and Computer Research.* 12, 1, 63-70, (2016).
2. Bhat M. A., Raja T. A., Maqbool S., Sofi N. A., Rauf A. B., Baba S. H., and Immad A. S., Robust Estimators for Estimation of Population Variance Using Linear Combination of Downton’s Method and Deciles as Auxiliary information. *Advances in Research.* 15,2, 1-7, (2018). Article no. AIR.41956 (DOI:10.9734/AIR/2018/41956),
3. Evans W. O., On the Variance of Estimates of the Standard Deviation and Variance. *Journal of Americal Statistical Association,* 46, 220-224, (1951).

4. Gupta S., and Shabbir J., Variance Estimation in Simplr random sampling using Auxiliary information. Hacettepe *Journal of Mathematics and Statistics.* 37, 57-67, (2008).
5. Hansen M. and H., Hurtwitz W. N., Sample Survey Methods and Theory Wiley, New York, (1953).
6. Isaki C. T., Variance Estimation using Auxiliary information. *Journal of the American Statistical Association.* 78, 117-123, (1983).
7. Kadilar C., and Cingi H., Improvement in Variance Estimation using Auxiliary informatio. *Hacettepe Journal of Mathematics and Statistics.* 35, 111-115, (2006).
8. Khan M., and Shabbir J. A., Ratio Type Estimator for the Estimation of Population Variance using Quartiles of an Auxiliary Variable. *Journal of Statistics Applications and Probability.* 2, 3, 319-325, (2013).
9. Maqbool S., and Shakeel J., Variance Estimation Using Linear Combination of Tri-mean and Quartile Average. *American Journal of Biological and Environmental Statistics*. 3,1,5-9, (2017)
10. Muili J. O., Agwamba E. N., Erinola Y. A., Yunusa M. A., Audu A., and Hamzat M. A., A Family of Ratio-Type Estimators of Finite Population Variance. *International Journal of Advances in Engineering and Management.* 2, 4, 309-319, (DOI: 10.35629/5252-0204309319) (2020).
11. Muili J. O., Audu A., Singh R. V. K., and Yunusa I. A., “Improved Estimators of Finite Population Variance Using Unknown weight of Auxiliary Variable. *Annals Computer Science Series.* 17, 1, 148-153, (2019).
12. Muili J. O., Singh R. V. K., and Audu A., Study of Efficiency of Some Finite Population Variance Estimators in Stratified Random Sampling. *Continental Journal of Applied Sciences.* 13, 2, 1-17, (2018).
13. Murthy M. N., Sampling Theory and Method. *Calcutta Statistical Publishing House,* India, (1967).
14. Prasad B., and Singh H. P., Some improved Ratio-Type Estimators of finite Population Variance using Auxiliary information in sample surveys. *Communication in Statistics Theory and Methods.* 19, 3, 1127-1139, (1990).
15. Subramani J., Generalized Modified Ratio Type Estimators for Estimation Population Variance. *Sri-lankan Journal of Applied Statistics*. 16,1, 69-90, (2015).
16. Tailor R., and Sharma B., Modified Estimators of Ppoulation Variance in the presence of Auxiliary informatio. *Statistics in Transition-New series.* 13, 1, 37-46, (2012).
17. Yadav S. K., Mishra S. S., and Gupta S., Improved Variance Estimation Utilizing Correlation Coefficient and Quartiles of an Auxiliary Variable. *Communicated to American Journal of Mathematics and Mathematical Sciences,* (2014).