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Logarithmic-Product-Cum-Ratio Type Estimator for Estimating Finite Population Coefficient of Variation

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Abstract

Estimation of population parameters have been a challenging aspect in sample survey for sometimes and many efforts have been made to enhance the exactness of the parameters of these estimators. We suggested the logarithmic-product-cum-ratio type estimator. Expression of the MSE of the intended estimator originated using the Taylor series technique. A numerical illustration was conducted and the results revealed that the modified work is ameliorated than sample means with other estimators observed.

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Introduction

Statistically, the coefficient of variation is used to make a comparison between two or more things with different units or dimensions, for example,the weight and height of individuals, are in kilograms (kg) and centimeters (cm). Auxiliary variables are often used in sample surveys to obtain improved precision in the estimate of the population characteristics of the study variable. This information may be used at both the design and estimation stages. Ratio, regression, and product method of estimation are used in this setting. For appraising population parameters of the study variable using the information on auxiliary variables, several researchers[3,4,9,11,12,13,15,16 and17] have performed a considerable amount of work Those of them ceased to emphasize the difficulty of estimating the coefficient of variation for a long time. But some still do, like[1] first suggested the estimator for the coefficient of variation when samples were selected using SRSWOR while other work includes.[2,6,7,8,10,18,19 and 20]

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In this current study, a class of logarithmic-product-cum-ratio type estimatorsfor the estimation of the coefficient of variation have been proposed. The proposed class of estimators is expected to produce an estimate nearer to the accurate population coefficient of variation.

Methodology

Let us consider a simple random sample n drawn from the given population of N units. Let the value of the study variable Y and the auxiliary variable X for the i^{th} units (i=1, 2,3, 4, …, N) of the population be denoted by Y_i and X_i for the i^{th} unit in methods: population means of the auxiliary variables, population variance of the study variable, population variance of the auxiliary variable, population covariance of the auxiliary and study variable, *p* denotes the correlation coefficient between X and Y.

$$\bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i,\ \bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i,\ s_y^2=\frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2$$

$$s_x^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2,\ C_y=\frac{S_y}{\bar{Y}}$$

$$s_{xy}=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y}),\ \bar{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i,\ \bar{X}=\frac{1}{N}\sum_{i=1}^{N}X_i,$$

$$\gamma=\frac{(1-f)}{n},\ f=\frac{n}{N},$$

$$S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\bar{Y})^2,\ S_x^2=\frac{1}{N-1}\sum_{i=1}^{N}(X_i-\bar{X})^2,$$

$$S_{xy}=\frac{1}{N-1}\sum_{i=1}^{N}(X_i-\bar{X})(Y_i-\bar{Y}),$$

$$C_x=\frac{S_x}{\bar{X}},,\ \lambda_{rs}=\frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}$$

Existing Estimators

The sample coefficient of variationis given as,

$$t_0=\hat{C}_y=\frac{s_y}{\bar{y}} \quad \text{…(1)}$$

The variance is given by.

$$MSE(t_0)=C_y^2\gamma\left(C_y^2+\frac{1}{4}(\lambda_{40}-1)-C_y\lambda_{30}\right) \quad \text{…(2)}$$

[2]ratio estimator of population coefficient of variation of the study variable utilizing information on the population mean.

$$t_{AR1}=\hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right) \quad \text{…(3)}$$

$$MSE(t_{AR1})=C_y^2\gamma\left[C_y^2+\frac{1}{4}(\lambda_{40}-1)+C_x^2-C_x\lambda_{21}-C_y\lambda_{30}+2\rho C_yC_x\right] \quad \text{…(4)}$$

They also proposed a ratio estimator of the population coefficient of variation of the study variable utilizing information on the population variance.

$$t_{AR2}=\hat{C}_y\left(\frac{S_x^2}{s_x^2}\right) \quad \text{…(5)}$$

$$MSE(t_{AR2})=C_y^2\gamma\left[C_y^2+\frac{1}{4}(\lambda_{40}-1)+(\lambda_{04}-1)-(\lambda_{22}-1)-C_y\lambda_{30}+2C_y\lambda_{12}\right] \quad \text{…(6)}$$

[18]the following estimators for the coefficient of variation based on information on a single auxiliary variable, the population mean, as

$$t_1=\hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha} \quad \text{…(7)}$$

$$t_2=\hat{C}_y\exp\left\{\beta\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right\} \quad \text{…(8)}$$

$$t_3=\ddot{C}_y+d_1\left(\bar{X}-\bar{x}\right) \quad \text{…(9)}$$

The MSE expressions of the estimators are given by,

$$MSE(t_1)=C_y^2\gamma\left[C_y^2+\frac{\lambda_{40}-1}{4}+\alpha^2C_x^2-C_y\lambda_{30}+2\rho C_yC_x-\alpha C_x\lambda_{21}\right] \quad \text{…(10)}$$

$$MSE(t_2)=C_y^2\gamma\left[C_y^2+\frac{\lambda_{40}-1}{4}+\frac{\beta^2C_x^2}{4}-C_y\lambda_{30}+\beta\rho C_yC_x-\frac{\beta}{2}C_x\lambda_{21}\right] \quad \text{…(11)}$$

$$MSE(t_3)=\gamma\left[C_y^2\left(C_y^2-C_y\lambda_{30}+\frac{\lambda_{40}-1}{4}\right)+d_1^2\bar{X}^2C_x^2+2d_1\bar{X}\rho C_y^2C_x-d_1\bar{X}C_yC_x\lambda_{21}\right] \quad \text{…(12)}$$

Where $\alpha=\frac{\lambda_{21}-2\rho C_y}{2C_x}$, $\beta=\frac{\lambda_{21}-2\rho C_y}{C_x}$, $d_1=\frac{C_y\lambda_{21}-2\rho C_y^2}{2\bar{X}C_x}$.

[18]the following estimators for coefficient of variation based on information on single auxiliary variable, population variance, as

$$t_4=\hat{C}_y\left(\frac{S_x^2}{s_x^2}\right)^{\alpha} \quad \text{…(13)}$$

$$t_5=\hat{C}_y\exp\left\{\beta\left(\frac{S_x^2-s_x^2}{S_x^2+s_x^2}\right)\right\} \quad \text{…(14)}$$

$$t_6 = \hat{C}_y + d_2\left(S_x^2 - s_x^2\right) \quad \ldots(15)$$

The mean square error (MSE) expressions of the estimators are given by.

$$MSE(t_4) = C_y^2\gamma\left[C_y^2 + \frac{\lambda_{40}-1}{4} + \alpha^2(\lambda_{04}-1) - C_y\lambda_{30} + 2\alpha C_y\lambda_{12} - \alpha(\lambda_{22}-1)\right] \quad \ldots(16)$$

$$MSE(t_5) = C_y^2\gamma\left[C_y^2 + \frac{\lambda_{40}-1}{4} + \frac{\beta^2(\lambda_{04}-1)}{4} - C_y\lambda_{30} + \beta C_y\lambda_{12} - \frac{\beta}{2}(\lambda_{22}-1)\right] \quad \ldots(17)$$

$$MSE(t_6) = \gamma\left[\begin{array}{l} C_y^2\left(C_y^2 - C_y\lambda_{30} + \frac{\lambda_{40}-1}{4}\right) + 2C_y^2 d_2 S_x^2\lambda_{12} + d_2^2 S_x^4(\lambda_{04}-1) \\ -C_y d_2 S_x^2(\lambda_{22}-1) \end{array}\right] \quad \ldots(18)$$

Where $\alpha = \frac{\lambda_{22}-1-2C_y\lambda_{12}}{2(\lambda_{04}-1)}, \beta = \frac{\lambda_{22}-1-2C_y\lambda_{12}}{(\lambda_{04}-1)}, d_2 = \frac{C_y(\lambda_{22}-1)-2C_y^2\lambda_{12}}{2S_x^2(\lambda_{04}-1)}$.

[20]logarithmic ratio type estimator for the estimation of population coefficient of variation when the natural logarithm of the auxiliary variable is known as,

$$T_y = \hat{C}_y\left(\frac{Ln(S_x^2)}{Ln(s_x^2)}\right) \quad \ldots(19)$$

The mean square error (MSE) of (19) is given by,

$$MSE(T_y) = C_y^2\gamma\left(C_y^2 + \frac{(\lambda_{40}-1)}{4} + \frac{(\lambda_{04}-1)}{\left(Ln(S_x^2)\right)^2} - \frac{(\lambda_{22}-1)}{Ln(S_x^2)} - C_y\lambda_{30} + \frac{2C_y\lambda_{12}}{Ln(S_x^2)}\right) \quad \ldots(20)$$

Proposed Estimator

Motivated by the work of,[20] we suggested a logarithmic-product-cum-ratio type estimator for estimating the coefficient of variation as,

$$T_{am} = \hat{C}_y\left(\frac{Ln(\bar{x})}{Ln(\bar{X})}\right)\left(\frac{Ln(S_x^2)}{Ln(s_x^2)}\right) \quad \ldots(21)$$

The above estimator is defined under the assumptions that,

$$e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}, e_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}, e_2 = \frac{s_y^2 - S_y^2}{S_y^2}, e_3 = \frac{s_x^2 - S_x^2}{S_x^2}$$

Such that

$$\bar{y} = \bar{Y}(1+e_0), \bar{x} = \bar{X}(1+e_1), s_y = S_y(1+e_2)^{1/2}, s_x = S_x(1+e_3)^{1/2},$$

$$s_y^2 = S_y^2(1+e_2), s_x^2 = S_x^2(1+e_3)$$

$$E(e_0) = E(e_1) = E(e_2) = E(e_3) = 0$$

$$E(e_0^2) = \gamma C_y^2, E(e_1^2) = \gamma C_x^2, E(e_2^2) = \gamma(\lambda_{40}-1), E(e_3^2) = \gamma(\lambda_{04}-1),$$

$$E(e_0e_1) = \gamma\rho C_yC_x, E(e_0e_2) = \gamma C_y\lambda_{30}, E(e_0e_3) = \gamma C_y\lambda_{12},$$

$$E(e_1e_2) = \gamma C_x\lambda_{21}, E(e_1e_3) = \gamma C_x\lambda_{03}, E(e_2e_3) = \gamma(\lambda_{22}-1)$$

Derivation of the Proposed Estimator T_{am}

Expressing (21) in terms of error terms defined in section 1, we have,

$$T_{am} = \frac{S_y(1+e_2)^{1/2}}{\bar{Y}(1+e_0)}\left(\frac{Ln(\bar{X}(1+e_1))}{Ln(\bar{X})}\right)\left(\frac{Ln(S_x^2)}{Ln(S_x^2(1+e_3))}\right) \quad \ldots(22)$$

Expand (22) using law of logarithm, we obtained (23)

$$T_{am} = \frac{S_y(1+e_2)^{1/2}}{\bar{Y}(1+e_0)}\left(\frac{Ln(\bar{X})+Ln(1+e_1)}{Ln(\bar{X})}\right)\left(\frac{Ln(S_x^2)}{Ln(S_x^2)+Ln(1+e_3)}\right) \quad \ldots(23)$$

Simplifying (23) by factorizing $\bar{X}$ and $Ln(S_x^2)$ from the numerator and denominator of the expression in the last two brackets of (23), we have obtained (24)

$$T_{am} = C_y(1+e_2)^{1/2}(1+e_0)^{-1}(1+\theta_1 Ln(1+e_1))\left(\frac{1}{1+\theta_2 Ln(1+e_3)}\right) \quad \ldots(24)$$

$$T_{am} = C_y(1+e_2)^{1/2}(1+e_0)^{-1}(1+\theta_1 Ln(1+e_1))(1+\theta_2 Ln(1+e_3))^{-1} \quad \ldots(25)$$

where, $\theta_1 = 1/Ln(\bar{X}), \theta_2 = 1/Ln(S_x^2)$

By expanding $Ln(1+e_3)$, $(1+e_2)^{1/2}$, $Ln(1+e_1)$ and $(1+e_0)^{-1}$ up to first order of approximation we have,

$$T_{am} = C_y\left(1+\frac{e_2}{2}-\frac{e_2^2}{8}\right)(1-e_0+e_0^2)\left(1+\theta_1\left(e_1-\frac{e_1^2}{2}\right)\right)\left(1-\theta_2\left(e_3-\frac{e_3^2}{2}\right)+\theta_2^2e_3^2\right) \quad \ldots(26)$$

$$T_{am} = C_y\left(1-e_0+e_0^2+\frac{e_2}{2}-\frac{e_0e_2}{2}-\frac{e_2^2}{8}\right)\left(1+\theta_1e_1+\frac{\theta_1e_1^2}{2}\right)\left(1-\theta_2e_3+\left(\theta_2^2+\frac{\theta_2}{2}\right)e_3^2\right) \quad \ldots(27)$$

Simplify, Subtract from both sides and consider terms of degree one, we have

$$T_{am} - C_y = C_y\left(\frac{e_2}{2} - e_0 + \theta_1e_1 - \theta_2e_2\right) \quad \ldots(28)$$

Square both sides of equation (28) and expand right hand side relation to first order of approximation we have,

$$(T_{am}-C_y)^2 = C_y^2\left(\begin{array}{l}\frac{e_2^2}{4} + e_0^2 + \theta_1^2e_1^2 + \theta_2^2e_3^2 - e_0e_2 + \theta_1e_1e_2 - \theta_2e_2e_3 - 2\theta_1e_0e_1 \\ +2\theta_2e_0e_3 - 2\theta_1\theta_2e_1e_3\end{array}\right) \quad \ldots(29)$$

Take expectation on both sides of equation (29) to obtain the MSE of the proposed estimator as:

$$MSE(T_{am}) = C_y^{\,2}\gamma\left(\begin{array}{l}\frac{(\lambda_{40}-1)}{4} + C_y^{\,2} + \theta_1^2 C_x^2 + \theta_2^2(\lambda_{04}-1) - C_y\lambda_{30} + \theta_1 C_x\lambda_{21} - \theta_2(\lambda_{22}-1) \\ -2\theta_1\rho C_y C_x + 2\theta_2 C_y\lambda_{12} - 2\theta_1\theta_2 C_x\lambda_{03}\end{array}\right) \quad \ldots(30)$$

Numerical Analysis

A numerical analysis was carried out to clarify the accomplishment of proposed estimator. Data used are from the book[5] and.[14]

Dataset 1

X: Area under wheat in 1963, Y: Area under wheat in 1964

N=34, n=15, $\bar{X}$=208.88, $\bar{Y}$=199.44, C_x=0.72, C_y=0.75, p=0.98, λ_{21}=1.0045, λ_{12}=0.9406, λ_{40}=3.6161, λ_{04}=2.8266, λ_{30}= 1.1128, λ_{03} =0.9206, λ_{22} =3.0133

Dataset 2

X: Number of fish caught in year 1993, Y: Number of fish caught in year 1995

N=69, n=40, $\bar{X}$ =4591.07, $\bar{Y}$=4514.89, C_x=1.38, C_y=1.35, p=0.96, λ_{21}=2.19, λ_{12}=2.30, λ_{40}=7.66, λ_{04}=9.84, λ_{30}= 1.11, λ_{03} =2.52, λ_{22}=8.19

Table1 above revealed that mean square error of the suggested estimator is minimal compared to those estimators considered in this study. This implies that the suggested estimator is ameliorated, more so, this is evidence, as the proposed estimator has a higher percentage of relative efficiency.

Table 1: MSE and Percentage Relative Efficiencies of proposed estimator and the existing estimators.

ESTIMATORS	DATASET 1		DATASET 2	
	MSE	PRE	MSE	PRE
t_0	0.0080036	100.00	0.038088	100.00
t_{AR1}	0.0258907	30.91	0.0851798	44.71
t_{AR2}	0.0336578	23.78	0.18860299	20.19
t_1	0.006868341	116.53	0.03731461	102.07
t_2	0.006868341	116.53	0.03731461	102.07
t_3	0.006868341	116.53	0.03731461	102.07
t_4	0.006962763	114.95	0.037568156	101.38
t_5	0.006962763	114.95	0.037568156	101.38
t_6	0.006962763	114.95	0.037568156	101.38
T_Y	0.0071255	112.32	0.0375686	101.38
T_{am}	0.005672477	141.10	0.03591946	106.04

Results and Discussion

An efficient product-cum-ratio of finite population coefficient of variation is proposed. The attribute of the proposed estimator was secured. The results hows the mean square errors and percentage relative efficiencies of the suggested estimator and some existing work using the two data sets. The results revealed that the proposed estimator is more efficient than other existing estimators considered in the study.

Conclusion

Currently, we suggested logarithmic-product-cum-ratio type estimator for estimating the coefficient of variation of the study variable, and this estimator employed information on the natural logarithm of the sample and population mean as well as the sample and population variance of the auxiliary variable. From numerical analysis, the results show that the proposed estimator is more efficient than other existing estimators considered in the study. Hence, the ameliorated proposed estimatoris recommended for practical usage.

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Conflict of Interest

The authors declare no conflict of interest.

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