



Neutrosophy Vs Fuzziness in Assessment Processes

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Abstract

In this paper Neutrosophic Sets are used as tools for assessing the overall performance of a group, when the individual performance of its members (e.g. students, players, etc.) is uncertain. Examples are also given to illustrate our results



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The theory of *Neutrosophy*, introduced by Smarandache¹ in 1995, is an extension of the Zadeh's *Fuzziness*² In a *neutrosophic set* (NS) all the elements of the universal set are characterized by three parameters, which take values in the unit interval $[0, 1]$. These parameters are the degree m of *membership (or truth)* as in the *fuzzy sets* (FSs), and in addition the degrees n of *non-membership (or falsity)* and i of *indeterminacy (or neutrality)*. In other words, each element of a NS can be written in the form of a *neutrosophic triplet* (m, i, n) , with $0 \leq n+i+m \leq 3$.³ To be more precise, this is the simplest form of a NS, usually referred to as a single valued NS.³ Notice that the degree of non-membership was introduced earlier by Atanassov when extending the

concept of fuzzy set to the concept of *intuitionistic FS*.⁴ Smarandache¹ introduced the degree of indeterminacy later, inspired by the frequently appearing in everyday life neutralities, like <short, medium, high>, <win, draw, defeat>, etc.

For example, if a football player X is characterized by the triplet $(0.7, 0.2, 0.4)$ in the NS of the good players, this means that there is a 70% belief that X is a good player; at the same time, however, there is a 40% belief that X is not a good player and a 20% doubt whether he is a good player or not These data are of course inconsistent, but this happens frequently in everyday life situations, since human reasoning is often characterized by inconsistency.

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In general, if $n+i+m < 1$, then we speak about data leaving room to incomplete information, if $n+i+m = 1$ to complete and if $n+i+m > 1$ about data leaving room to inconsistent information. ANS may simultaneously contain elements leaving room to all these kinds of information.

It is of worth mentioning that there is no general rule for defining the degrees of membership, non-membership and indeterminacy of the elements a NS, the methods used about this being usually statistical or empirical based on each observer's personal criteria. This is a serious disadvantage of the concept of FS and of all its extensions involving membership degrees. Several efforts have been made by the researchers to overcome this difficulty by introducing the *interval valued FSs*,⁵ *the type – n FSs*, $n \geq 2$,⁶ *the rough sets*,⁷ *the soft sets*,⁸ *the grey system theory*,⁹ etc.

NSs have found important applications to practical everyday life problems (e.g. for assessment,¹⁰ decision-making,¹¹ etc.), in which the use of Zadeh's FSs has been proved not efficient for obtaining the required results. The applications of NSs have been also extended to areas of classical mathematics, like Algebra, Geometry, Analysis, Topology, etc. (e.g. see,)^{12,13} The purpose of this short communication is to sketch the application of NSs to assessment processes.

In fact, frequently the performance of the members of a group is assessed by using qualitative grades (linguistic expressions) instead of numerical ones. This happens either because the existing data about their performance are not very clear, or for reasons of elasticity (e.g. from teacher to students). Obviously, in such cases the mean performance of the group cannot be assessed by calculating the mean value of the individual scores of its members. For tackling this situation, we have used in earlier works either *triangular fuzzy numbers* (TFNs) or *grey numbers* (GN's) - i.e. closed real intervals - and we have shown that these two methods are equivalent¹⁴ (sections 5.2 and 6.2).

Cases appear, however, in practice, in which one is not sure about the accuracy of the qualitative grades assigned to the objects under assessment (e.g. students). In such cases the use of TFNs or GNs is

not possible and the use of NSs looks as the best way for evaluating the group's overall performance. In the following example, which illustrates this situation, the *addition* between *neutrosophic* triplets and the *scalar product* of a positive number with a *neutrosophic* triplet are defined as in the ordinary ordered triplets, i.e. by

$$(m_1, i_1, n_1) + (m_2, i_2, n_2) = (m_1+m_2, i_1+i_2, n_1+n_2) \dots(1)$$

and

$$k(m_1, i_1, n_1) = (km_1, ki_1, kn_1) \dots(2),$$

for all m_s, i_s, n_s in $[0, 1]$, $s=1, 2$ and all $k > 0$. We must note that the previous operations are not closed operations between *neutrosophic* triplets, since we may have that

$$(m_1+m_2) + (i_1+i_2) + (n_1+n_2) > 3 \text{ or } km_1 + ki_1 + kn_1 > 3.$$

They can be used, however for defining the mean value of a finite number of *neutrosophic* triplets of a NS as follows:

Let $(m_1, i_1, n_1), (m_2, i_2, n_2), \dots, (m_k, i_k, n_k)$ be a finite number of elements of a NS. Then their mean value is defined to be the element (m, i, n) of this NS calculated by

$$(m, i, n) = 1/k (m_1, i_1, n_1) + (m_2, i_2, n_2) + \dots + (m_k, i_k, n_k) \dots(3)$$

Example

The new manager of a company is not sure yet about the quality of his employees. He decided, therefore, based on his personal criteria, to characterize the set of the very good employees by *neutrosophic* triplets as follows: $e_1(1, 0, 0)$, $e_2(0.9, 0.1, 0)$, $e_3(0.8, 0.1, 0.1)$, $e_4(0.7, 0.2, 0.1)$, $e_5(0.7, 0.1, 0.2)$, e_6, e_7 and e_8 by $(0.6, 0.2, 0.2)$, e_9 and e_{10} by $(0.5, 0.3, 0.2)$, $e_{11}(0.4, 0.4, 0.2)$, e_{12}, e_{13} and e_{14} by $(0.3, 0.3, 0.4)$, $e_{15}(0.1, 0.5, 0.4)$ and e_{16} by $(0, 0, 1)$. This means that the manager of the company is absolutely sure that e_1 is a very good employee, 90% sure that e_2 is a very good employee too, but at the same time he has a 10% doubt about it, 80% sure that e_3 is a very good employee having a 10% doubt about it and simultaneously a 10% belief that e_3 is not a very good employee, and so on.. For the last employee

e16 the manager is absolutely sure that he is not a very good employee. How can he estimate the mean level of the employees' skills?

Solution

The mean level of the employees' skills can be estimated by the mean value of the given *neutrosophic* triplets, i.e. by $1/6 (1, 0, 0)+(0.9, 0.1, 0)+(0.8, 0.1, 0.1)+(0.7, 0.2, 0.1)+(0.7, 0.2, 0.1)+3(0.6, 0.2, 0.2)+2(0.5, 0.3, 0.2)+(0.4, 0.4, 0.2)+3(0.3, 0.3, 0.4)+(0.1, 0.5, 0.4)+(0,0,1)$, which by equations (1) and (2) is equal to $1/16 (8.3, 3.6, 4.1) \approx (0.519, 0.225, 0.256)$. This means that a random employee of the company is by 51.9 % a very good employee, but there exists also a 22.5% doubt about it and a 25.6% belief that he is not a very good employee.

Conclusion

On the basis of the discussion performed in this article, one concludes that the advantage of using NSs as tools for assessment is that it enables the

evaluation of a group's overall performance in cases where the assessment data are uncertain. On the contrary, the main weakness of this method is that it gives an approximate idea of the group's overall performance only, not being able to provide an exact measure for its mean performance, as the use of grey or fuzzy numbers, when it is possible, does (see,¹⁴ sections 5.2 and 6.2).

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Conflict of Interest

The authors do not have any conflict of interest.

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